Shock Wave of Explosion

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Introduction

The personnel is the most important objective of exploding ammunitions and the primary target of military actions.

Blast ammunitions and explosive charges are constructed to incapacitate a large number of personnel (soft targets) and hard targets (constructions, armament, guns, shelters, etc.) by means of blast wave of explosive charges, artillery and aviation projectiles, etc.

The blast ammunitions have a relatively large explosive charge (TNT, RDX, C-4, etc.) necessary to fragment the metallic shell and create the blast needed to incapacitate the military personnel and hard targets through the destruction action of **shock wave** (SW). In general, for blast ammunitions with metallic shells, the lethal range of SW of the explosive charge is relatively larger compared to the lethal range of the fragments delivered by the same blast ammunition.

For more information, the reader can see [2].

1. Detonation of Explosives.

The process of expansion of chemical reactions that transforms the chemical explosive charge into gaseous products of high density, temperature and pressure is called detonation, or detonation wave. The detonation of a high explosive charge, HE, in each direction, is result of the shock wave which, after initiation by a detonator, keeps the chemical reaction to flow, without interruption, with a constant supersonic velocity.

The detonation wave is practically a shock wave (SW) which travels with high velocity in all directions, compressing vigorously the solid explosive and triggering the chemical reactions in the solid explosive charge.

The velocity of detonation for a given kind of explosive is constant, some thousand meters per second. As result of that high velocity, at the end of detonation the gaseous products occupy almost the same volume as the explosive charge (see Instantaneous Detonation, p.10). The detonation velocity does not depend on the characteristics of the surrounding atmosphere, other constructions, projectile shell, etc. The detonation wave ends at the time when all the explosive charge is transformed in gaseous products of high pressure, temperature and density. Gaseous products expanding in stationary air create the air shock wave (blast wave).

To study the SW in air (gas, explosive charge, solid body, water, etc.) the Gasdynamics/Hydrodynamics considers the physical model of a tube with a fast-moving piston.

Note that, the Gasdynamics theory is not object of this paper. The theoretical outcomes are presented without demonstrations, but are illustrated mostly with examples.

To simplify the study of air shock wave in Gasdynamics it is used the simplified Physical model of instantaneous detonation, i.e. we assume that the explosive charge is transformed instantly in gaseous products at the moment the chemical reactions initiate at the location of a detonator.

1.1 Detonation Shock Wave

Important characteristics of detonation are the parameters of detonation (pressure, density, velocity of particles) at the front of detonation SW. They can be estimated using respectively the corresponding approximate formulae:

$$P_{H=\rho_0 \cdot D^2/4} \ , \quad \rho_{H=(4/3)\cdot}\rho_0, \qquad u_H = D/4 \ , \tag{1.1}$$

where ρ_0 is the density of explosive charge, D is the velocity of detonation (Fig. 1).

The above formulas can be used, with a good approximation, to solve practical problems.

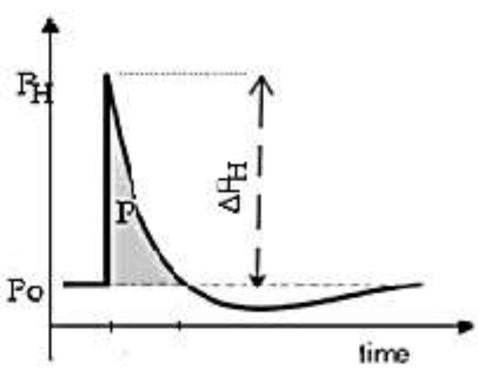


Fig. 1

At the front of SW of detonation, the pressure, density and temperature jump quite instantly. Behind the front there is a thin layer where chemical reactions create the gaseous products that move behind the front of SW.

In Fig. 1 it is shown the detonation wave, at a certain distance r from the center of explosion. Detonation wave is composed by the positive pressure phase (compressed zone) and the negative one (rarefaction zone). At positive pressure phase the gas particles move after the front of the SW. In rarefaction part of the SW the particles of gases move in the opposite direction.

At the front of the detonation shock wave the pressure jumps almost immediately from the value of the atmospheric pressure $P_0 = P_{atm}$ to the maximum pressure P_H. The difference $\Delta P_H = P_H - P_0$ is the maximum overpressure (pick overpressure).

The parameters at the front of detonation shock wave are given in formulae (1.1).

1.2 Detonation Velocity

The detonation velocity of an explosive charge with a given density greater than 1000 kg/m² can be calculated using the experimental formula of M. A. Kuk [1, p. 132]:

$$D = D_0 + M(\rho - \rho_0) \tag{1.2}$$

where D and ρ are respectively the velocity of detonation and the density of explosive mass, while D_0 is the velocity of detonation that corresponds to the density ρ_0 of the explosive charge, and M is a coefficient that depends on the characteristics of explosive (see Table 1).

Thus, for example, using formula (1.2) and the data from table 1, for the TNT charge with density $\rho = 1550\ kg/m^3$, we find the detonation velocity,

$$D = D_0 + M(\rho - \rho_0) = 5010 + 3.225 \cdot (1550 - 1000) = 6784\ m/s.$$

Thus, for example, for the TNT explosive charge with density $\rho = 1617 kg/m^3$, we find that the detonation velocity is:

$$D = 5010 + 3.225 \cdot (1550 - 1000) = 7000\ m/s\ ,$$

Table 1.

	$\rho_0\ [\frac{kg}{m^3}]$	$D_0[\frac{m}{s}]$	M $[(\frac{m}{s})/(\frac{kg}{m^3})]$
TNT	1000	5010	3.225
TEN	1000	5550	3.950
PENTOLIT 50/50	1000	5480	3.100
TETRYL	1000	5600	3.225
Hexogen -TNT 64/36	1600	7540	3.080

Note that the detonation velocity depends on the density of explosive. A higher density of a given unit of HE charge generates a higher amount of energy than the same explosive of unit charge of smaller density.

For that reason, to have a standard TNT charge, it is considered as reference the density $\rho_0 = 1600\ kg/m^3$.

For such high velocities of detonation, i.e. high velocities of expansion of chemical

reactions, at the time when the detonation process comes to an end, the gaseous products practically have the same volume as the volume of explosive charge itself.
As result of the very small volume of the high-density gases, the pressure is very large, and so, the gases expand rapidly acting powerfully on the surrounding environment.

Note that density of gases at the instant of detonation is the same as the density of explosive charge; for the TNT standard charge the density is $\rho_0 = 1600\ kg/m^3$.
During the detonation process, the energy that is delivered by the chemical reactions allows, without interruption, the development of chemical reactions in all the volume of explosive. The high velocity of detonation practically does not let the gaseous products to expand before the detonation ends (see examples below).

1.3 Thermal Energy of Detonation Products. TNT Equivalent Charge

During the detonation, the chemical reactions of molecules of explosive release thermal energy, Q.

If we denote e the specific thermal energy that is generated during the detonation of one unit mass of the explosive charge, then the TNT mass m of the given explosive charge, which generates the thermal energy Q (Joule)**,** is obtained by the equation

$$Q = m \cdot e. \tag{1.3}$$

To estimate the thermal energy of detonation products obtained from a standard TNT charge, it is considered that 1 kg of TNT generates an energy

$$e = 4.18 \times 10^6\ J\ /\ kg$$

Substituting e, we obtain the energy generated by the detonation of a TNT explosive charge of mass m (in kg), i.e.

$$Q = 4.18 \times 10^6 \cdot\ m. \tag{1.4}$$

TNT equivalent mass of a given high explosive is the mass of that TNT charge that will produce an energy equal to the energy released by the explosive under consideration.

To convert a mass m of any explosive charge to a TNT equivalent mass, we refer to Table 2.1, [2].

For example, let's assume a mass of TNT explosive charge, m_{TNT}. It produces an energy of

$$Q_{TNT} = 4.18 \times 10^6 \cdot\ m_{TNT}.$$

For the explosive charge HMX with mass m_{HMX} that produces the same energy as the standard TNT mass (m_{TNT}) we can write

$$Q_{HMX} = 5.68 \times 10^6 \cdot\ m_{HMX},$$

where $e_{HMX} = 5.68 \times 10^6 J$ is the energy delivered by 1 kg HMX, [2].

Since we consider $Q_{HMX} = Q_{TNT}$ from the above equations we have:

$$5.68 \times 10^6\ m_{HMX} = 4.18 \times 10^6 \cdot\ m_{TNTeq}.$$

Hence, the TNT equivalent charge of HMX is

$$m_{TNTeq} = (5.68 \times 10^6 / 4.18 \times 10^6) \cdot m_{HMX} = 1.36\ m_{HMX}.$$

Table 2 - Conversion Factors for Explosives

Explosive	**Mass Specific Energy** Qx **(kJ/kg)***	**TNT Equivalent** Q_X/Q_{TNT}
Compound B (60% RDX, 40% TNT)	5190	1.148
RDX (Cyclonite)	5360	1.185
HMX	5680	1.256
Nitroglycerin (liquid)	6700	1.481
TNT	4520	1.000
Pentolite	6012	1.330
60% Nitroglycerin Dynamite	2710	0.600
Semtex	5660	1.250

* Convert to (Btu lb.) by multiplying by 0.43

In the same way we find a general formula for the TNT equivalent mass (m_{TNTeq}) of any high explosive of mass m_e, i.e.

$$m_{TNTeq} = \left(\frac{Q_e}{4.18 \times 10^6}\right) \cdot m_e\ . \tag{1.5}$$

Note. There is a difference between the standard specific energy $e = 4.18 \times 10^6\ J/kg$ and the specific energy 4520 KJ/kg given in table 2. This is as result of the fact that the detonation velocity depends on the density of the HE explosive charge. Thus, the conversion factor of TNT equivalent mass, given in table 2, with respect to the standard TNT charge is

$$m_{TNTeq} = \left(\frac{Q_e}{4.18\times10^6}\right)\cdot m_e = \left(\frac{4520{,}000}{4.18\times10^6}\right)\cdot m_e = 1.08m_e.$$

while the density of the TNT in table 2 is $\rho_{01} = 1.08\cdot1600 = 1728\ kg/m^3$(See exercise below to understand how we find the density $\rho_{01} = 1728$).

Example 1.
Find the standard TNT equivalent mass of a TNT explosive charge with density $m = 1617\ kg/m^3$.

Solution:

Since the energy released during the detonation of an explosive charge is proportional to the density, we can write:

$$m_{TNTeq} = \left(\frac{1617}{1600}\right)\cdot m_e = 1.01\ kg.$$

1.4 Expansion of Detonation Products

To have an idea on the quantity of gaseous products of detonation, let's consider 1 kg. TNT charge with standard density $\rho_0 = 1600kg/m^3$, which occupies the volume of

$$V_0 = \frac{1}{\rho_0} = \frac{1}{1600} = 6.25\cdot10^{-4}\ m^3 \tag{1.6}$$

In general, if the mass of the explosive charge is **m** then the volume is

$$V = m/\rho. \tag{1.7}$$

where m and ρ are respectively the mass and the density of the explosive charge.

As we mention before, at the end of detonation process, the gaseous products (mass 1 kg, or mass **m** kg) will occupy almost the same volume as the explosive charge does. The pressure of detonation products, concentrated in such a small volume, is very high.

At the end of detonation process, **the average pressure $\boldsymbol{p_{av}}$** of the products of detonation, can be calculated using the following formula from the hydrodynamic theory of detonation:

$$p_{av} = P_H/2 = \rho_0 \cdot \frac{D_0^2}{8}. \quad (1.8)$$

Thus, for the TNT explosive charge with detonation velocity 6780 m/s, we have

$$p_{av} = \rho_0 \cdot \frac{D_0^2}{8} = 1600 \cdot \frac{6780^2}{8} = 9.19 \times 10^9 Pa = 9.38 \times 10^4\ KG/cm^2.$$

The Equation of State of the gaseous products, obtained during detonation, is

$$p \cdot V^3 = p_0 V_0^3 = constant \quad (1.9)$$

It can be written as well as

$$p/\rho^3 = p_0/\rho_0^3 = constant \quad (1.10)$$

The expansion of the gaseous products of detonation practically will "end" when the average pressure of expanding gases will fall till it becomes equal to the normal atmospheric pressure. In normal atmospheric conditions, pressure $p = 1\ atm = 1.013 \cdot 10^5 Pa = 1.033\ KG/cm^2$, temperature zero degree Celsius, the gaseous products

($volume\ \ V_0 = 6.25 \cdot 10^{-4}\ m^3$, pressure $p_0 = 1.013 \cdot 10^5 Pa$) will expand to have a large volume of

$$V = V_0 \cdot \sqrt[3]{\frac{P_{av}}{p_0}} = 6.25 \cdot 10^{-4} \cdot \sqrt[3]{\frac{9.20 \cdot 10^9}{1.013 \cdot 10^5}} = 0.0281\ m^3 \quad (1.11)$$

It means that high density gaseous products of detonation will expand to occupy a volume that is 46 times the volume $V_0 = (6.25 \cdot 10^{-4} m^3)$, of the undetonated explosive.

1.5 Spherical Standard TNT Charge and its Radius

Experiments show that the products of detonations of any concentrated explosive charge in any form, cube, sphere, rectangular, expands in the same way as a spherical charge of the same mass. That is not valid for a long cylindrical charge. Long linear explosive charge (LEC) is the one where one length is at least 4 times longer than the other lengths (E. Mori, [2]).

We can estimate the radius of the expansion of detonation products till the pressure will become equal to atmospheric value.

To simplify the solution of problems related to high explosive charges (HE), and to standardize those, we consider:

The mass **m** of a TNT spherical charge has the density 1600 kg/m³.
As we demonstrated above, the expansion of gases is "interrupted" when the average pressure of detonation gases become equal to the atmospheric pressure. At this moment, the SW, created as result of expansion of gases beyond the initial spherical explosive charge with radius r_0, separates itself from the detonation gases, though for inertia, the expansion of gases stops till around $20r_0$.
Let's estimate r_0.

Consider the mass m of a TNT spherical charge, radius r_0. We can write:

$$m = \frac{4}{3}\pi r_0^3 \cdot \rho_0 = \frac{4}{3}\pi r_0^3 \cdot 1600 \qquad (1.12)$$

since the density of TNT, $\rho_0 = 1600 kg/m^3$.

From the last equation, we find that the radius of the spherical charge, before initiation of detonation is

$$r_0 = (\frac{3}{4}\frac{m}{\pi \cdot 1600})^{1/3} = 0.053\sqrt[3]{m}. \qquad (1.13)$$

For example, a TNT charge of mass $m = 1\ kg\ TNT$ has the radius

$$r_0 = 0.053\sqrt[3]{1} = 0.053m.$$

This value appears in many writings on SWs of explosion.

Example 1.

For a mass m_{RDX}=2.5 kg of an explosive charge of RDX find the TNT equivalent mass of given RDX charge and the radius r_0, of the spherical charge.

Solution:
On Table 2.1 (E. Mori), is given that unit energy, $e_{RDX} = 5.36 \times 10^6 J/kg$. The TNT equivalent mass of RDX, i.e. m_{TNTeq}, is

$$m_{TNTeq} = \left(\frac{5.36 \times 10^6}{4.18 \times 10^6}\right) \cdot m_{RDX} = 1.282 m_{RDX} = 3.206\ kg\ TNT$$

Radius of the spherical RDX charge is

$$r_0 = 0.053\sqrt[3]{m_{TNT}}=0.053\sqrt[3]{3.206} = 0.0782\ m.$$

The expansion of the detonation products of a spherical charge continue till it "stops" when the average pressure becomes equal to the atmospheric pressure $p = 1.013 \times 10^5 P_a$ and the density equal to the density of air $\rho = 1.2929 kg/m^3$.

Let's estimate the distance where the detonation products stop expanding. Because the detonation products at the normal atmospheric condition are spherical, we can write

$$\mathrm{m}= \frac{4}{3}\pi \cdot r^3 \cdot \rho = \frac{4}{3}\pi \cdot r^3 \cdot 1.2929 \tag{1.14}$$

Since the mass of undetonated TNT explosive charge and the mass of gases of explosion are the same (no loss of mass), from two equations, (1.12) and (1.14), we have:

$$\frac{4}{3}\pi \cdot r^3 \cdot 1.2929 = \frac{4}{3}\pi r_0^3 \cdot 1600$$

Hence,

$$r = \sqrt[3]{1600/1.2929 \cdot}\, r_0 = 10.74 r_0 \approx 11 r_0 \tag{1.15}$$

Substituting (1.13) in (1.15), we find that the distance from the center of the explosive charge, where the detonation gases stop expanding is

$$r \approx 0.58 \cdot \sqrt[3]{m} \tag{1.16}$$

For an explosive charge of mass $m = 8\, kg\, TNT$ we find that the distance to the point where the SW is detached from the detonation gases is

$$r \approx 0.58 \cdot \sqrt[3]{8} = 1.16\, m$$

from the center of spherical charge.

Note that the separation distance r of the blast wave from the detonation gases can as well be easily found dividing equation (1.16) and (1.13), i.e.

$$r \approx 0.58 \cdot \sqrt[3]{m} \text{ and } r_0 = 0.053\sqrt[3]{m} \ :$$

$$\frac{r}{r_0} = \frac{0.58 \cdot \sqrt{m}}{0.053 \cdot \sqrt{m}} = 10.94 \approx 11$$

Hence

$$r = 10.94\, r_0 \approx 11\, r_0.$$

1.6 Instantaneous Detonation

Considering the detonation velocity $D = 7000\, m/s$, and a TNT charge of 1kg we find that the time when the detonation process ends is

$$t = r_0/D \approx 0.053 \cdot 1/7000 = 7.57 \times 10^{-6} \text{ sec.},$$

i.e. detonation occurs practically instantly at the time it is ignited.

That's why, as we mentioned above, in the hydrodynamic theory of shock waves, it is considered the model of instantaneous detonation of an explosive charge. The detonation gases at high pressure, and as result, at high density and high temperature, at the end of detonation process, have practically the same volume as that of the undetonated spherical explosive charge with radius $r_0 = 0.053\sqrt[3]{m}$.

Note: Shock wave in air and shock wave of detonation are different. The first one does not have the thin layer of chemical reaction; the velocity decreases continuously with the distance from center of explosion till it becomes sound wave; The SW deteriorate into a sound wave (there is no input of energy from any other source). On the other side, the velocity of SW of detonation is constant since the energy of chemical reactions keep the detonation velocity stable.

2. Shock Wave in Free Unconfined Air (Airburst)

2.1 Quantitative Characteristics of Shock Wave

The shock wave in air (blast wave), created as result of expansion of detonation gaseous, separates from the detonation products at the distance $r \approx 0.58 \cdot \sqrt[3]{m} = 11r_0$ from the center of the spherical charge. Because of the inertia, gaseous products continue to expand radially, from the center of the spherical charge, till at the distance $20r_0$. At this point, the gaseous products move backwards.

The blast wave, that is composed by the compressed phase of air and rarefaction phase (fig 2), travels with supersonic velocity D, which slows down to the speed of sound in air as result of energy loss. Note that at the rarefaction phase the pressure is below atmospheric pressure.

As the blast wave goes away from the center of explosion, part of energy of SW is dispersed over bigger and bigger spherical or semispherical surfaces. As result the specific energy diminishes. At the same time, the SW compresses and heats the atmospheric air in front of SW and as result it losses another part of energy. Thus, gradually, the velocity SW as well as all parameters of SW, become smaller and smaller. The SW degenerates to a sound wave, SW in air ceases to exist.

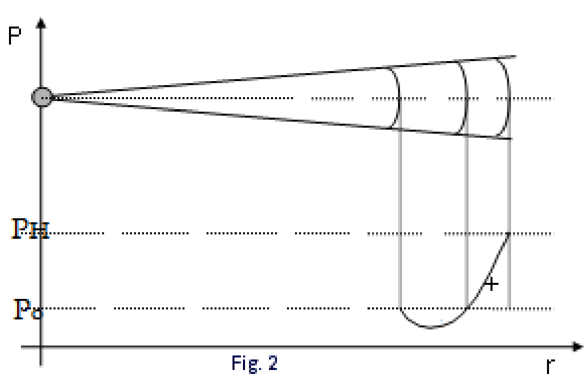
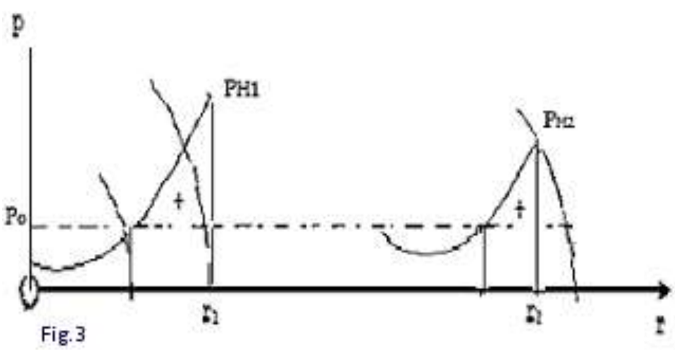


Fig. 2

In other words, the energy that the SW transports along any direction decreases with the increases of distance r from center (figure 3). The parameters at the front and in the positive pressure zone of the SW decrease with travel distance r.

The characteristics of SW are the parameters of air at the front of SW (pressure, temperature, density, velocity of particles of air), that change by jumping respectively from the values of stationary air P_0, T_0, ρ_0, u_0 to the values of compressed air, i.e. to the maximum (pick) values P_H, T_H, ρ_H, u_H.



As a matter of fact, the front of SW is a narrow zone through which occurs the change (jump) of values of parameters.

The thickness, ΔX [in cm], of the front of SW can be estimated by the formula

$$\Delta X = \frac{4\times10^{-5}}{\Delta P_H},$$

where $\Delta P_H = P_H - P_0$ is the hydrostatic overpressure at the front of SW measured in atmosphere.

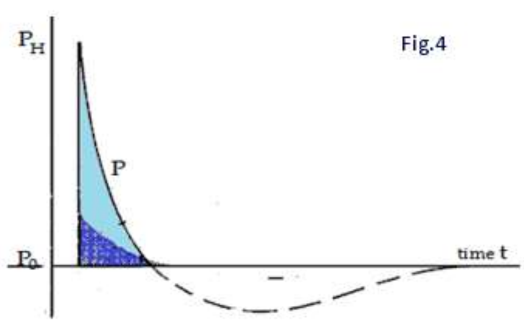
Thickness ΔX is of the order of the free range of molecules. Since that zone is too small the SW is treated as a dimensionless area where is realized the instantaneous change of parameters.

At this small zone the pressure increases from P_0 to P_H.

The time-interval τ of the positive phase is called the time-period of SW.

2.2 Maximum Overpressure

The lethal and destructive effects of an explosion, due to the shock wave, can be estimated using the static overpressure of SW, which is the difference of hydrostatic pressure P at a certain time t (Fig.4) and the pressure of atmospheric air, i.e.



$$\Delta P = P - P_0.$$

An indicator of the strength of the shock wave, that also serves to evaluate the level of injuries and devastating effects of blast on hard targets, is the Maximum Hydrostatic Overpressure at the front of shock wave (Fig. 4),

$$\Delta P_H = P_H - P_0$$

At a given distance r from the center of explosion, the overpressure, $\Delta P = P - P_0$, and the period τ (positive time-interval, or time-duration of SW, in seconds) can be estimated respectively by the formulas

$$\Delta P = \Delta P_H \cdot (1 - \frac{t}{\tau}) \cdot e^{-t/\tau}, \qquad (2.1)$$

and

$$\tau = 0.00115 \sqrt[6]{m} \cdot \sqrt[2]{r}\,, \qquad (2.2)$$

where t is the time, $0 \leq t \leq \tau$, m is the mass of explosive charge, and r is the distance of SW from the center of explosion.

Equation (2.1), shows that SW overpressure, at a certain distance r from the center of the explosive charge, decreases with time from the value of the peak overpressure ΔP_H (at time zero) to zero (when $t = \tau$).

Equation (2.2) shows that the time-interval τ of the positive phase, increases with the distance, while the peak overpressure decreases (see equations below).
The maximum overpressure ΔP_H (in MPa), at a distance r (meter) from the center of explosion in atmospheric air of a TNT charge with mass m (in kg), can be estimated using Sadovsky's experimental formula [4, page 478],

$$\Delta P_H = 0.87 \cdot \frac{\sqrt[3]{m}}{r} + 2.753 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 7.138 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3, \qquad (2.3)$$

where the maximum overpressure is expressed in KG/cm^2.

The max overpressure in (2.3) is valid for the explosion in unconfined atmospheric air when the peak-overpressure satisfies the condition:

$0.1\ KG/cm^2 \leq \Delta P_H \leq 10\ KG/cm^2$ or $1\ m/KG^{1/3} \leq \mathrm{r}/\sqrt[3]{m} \leq 10\ m/KG^{1/3}$. (2.4)

(We consider that $10.2\ kG/cm^2 = 1 MPa$).

The results obtained by Sadovsky's equation (2.3) are overrated.

A more accurate experimental formula to estimate ΔP_H , similar to Sadovsky's, is [4],

$$\Delta P_H = 0.79 \cdot (\frac{\sqrt[3]{m}}{r}) + 2.57 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3, \qquad (2.6)$$

valid for the pick overpressure that satisfies the condition:

$$0.1\, KG/cm^2 \leq \Delta P_H \leq 10\, KG/cm^2,$$

According to Orlenko, the equation (2.6) is valid for **any TNT explosive charge of mass** m.

The specific hydrostatic impulse of the positive phase of SW can be estimated by the formula:

$$i = \int_0^\tau \Delta P \cdot dt = 168\sqrt[3]{m^2}/r. \qquad (2.8)$$

(SI unit $P_a \cdot s = N \cdot s/m^2$).

Another important quantity of SW is the dynamic impulse of the flux of particles of air moving behind the front of SW, ((translational movement of gaseous particles).

It can be estimated using the practical equation:

$$j = \int_0^\tau \rho u^2 dt = 221 \cdot \sqrt[3]{m} \cdot (\frac{\sqrt[3]{m}}{r})^{2.5} \qquad \text{(Pa ·s).} \qquad (2.8a)$$

Note

- The equations/formulas we have used to estimate the max-overpressure and other important characteristics of SW are mainly those shown in references [2], [4].
- Equations (2.7), (2.8) and (2.8a) can be modified for surface ground explosion by substituting (2m) instead of (m), (see equations (3.9), (3.10) and (3.11)).

Example 1.

An explosive charge of mass m = 1 kg TNT explode in air. Find the time-interval τ at a distance $r = 2\ m$ from the center of explosion as well as the impulse of the positive phase i and the dynamic impulse j.

Solution

Using (2.7), (2.8) and (2.8a), we find:

$$\tau = 0.00115\sqrt[6]{m} \cdot \sqrt[2]{r} = 0.00115\sqrt[6]{1} \cdot \sqrt[2]{2} = 0.0016\ sec.$$

$$i = 168\sqrt[3]{m^2}/r = 168\sqrt[3]{1^2}/2 = 84 \text{ Pa}\cdot\text{s}.$$

$$j = 221 \cdot \sqrt[3]{m} \cdot (\frac{\sqrt[3]{m}}{r})^{2.5} = 221 \cdot \sqrt[3]{1} \cdot (\frac{\sqrt[3]{1}}{2})^{2.5} = 39.1 \text{ Pa } \cdot \text{s}.$$

2.3 **Another Set of Experimental Equations**

- $\Delta P_H = 25000\left(\frac{r_0}{r}\right)^3,$ (2.8b)

  valid for large strong SW. ($r_0 = 0.053\ \sqrt[3]{m}$)

  $\Delta P_H = 6.9 \cdot \left(\frac{m}{r^3}\right) + 1\ ,$ (2.9)

  valid for $\Delta P_H \geq 10\ KG/cm^2$.

- $\Delta P_H = 0.976 \cdot (\frac{\sqrt[3]{m}}{r}) + 1.50 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 6.04 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3 - 0.0196$ (2.10)

  valid for $0.1\ KG/cm^2 \leq \Delta P_H \leq 10\ KG/cm^2$.

We can use this equation when the mass of explosive charge is also $m \leq 100kg$. [4].

Note: According to John M. Dewey, 5% of the detonation products is lost as radiation [5].

Example 1.

Estimate the maximum overpressure at the front of the shock wave at a distance 30 meter from the center of explosion in air of a TNT explosive charge with mass:

1. $m = 30 \quad kg$.
2. $m = 200\ kg$.

Solution:

1. $m = 30kg, r = 30m$

(a) Using Sadovsky's formula (2.5)

Substituting in (2.5), $m = 30$, $r = 30$, we find:

$$\Delta P_H = 0.86 \cdot \frac{\sqrt[3]{30}}{30} + 2.75 \cdot \left(\frac{\sqrt[3]{30}}{30}\right)^2 + 7.14 \cdot \left(\frac{\sqrt[3]{30}}{30}\right)^3 = 0.127\ KG/cm^2.$$

(b) Using modified equation (2.6):

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{30}}{30}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{30}}{30}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{30}}{30}\right)^3 = 0.116\ KG/cm^2.$$

(c) Using (2.10), we have

$$\Delta P_H = 0.976 \cdot \left(\frac{\sqrt[3]{30}}{30}\right) + 1.500 \cdot \left(\frac{\sqrt[3]{30}}{30}\right)^2 + 6.04 \cdot \left(\frac{\sqrt[3]{30}}{30}\right)^3 - 0.0196 = 0.104\, KG/cm^2.$$

Note that $0.1 < \frac{\sqrt[3]{m}}{r} = \frac{\sqrt[3]{30}}{30} = 0.104 < 10$.

2. $m = 200kg, r = 30m$.

(a) Using Sadovsky's formula (2.5). Substituting $m = 200$, $r = 30$, we find:

$$\Delta P_H = 0.86 \cdot \frac{\sqrt[3]{200}}{30} + 2.75 \cdot \left(\frac{\sqrt[3]{200}}{30}\right)^2 + 7.14 \cdot \left(\frac{\sqrt[3]{200}}{30}\right)^3 = 0.325\ KG/cm^2.$$

(b) Using equation (2.6):

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{200}}{30}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{200}}{30}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{200}}{30}\right)^3 = 0.298\ KG/cm^2.$$

(c) Using (2.10), we have:

$$\Delta P_H = 1.007 \cdot \left(\frac{\sqrt[3]{200}}{30}\right) + 1.503 \cdot \left(\frac{\sqrt[3]{200}}{30}\right)^2 + 6.043 \cdot \left(\frac{\sqrt[3]{200}}{30}\right)^3 - 0.0196 = 0.278\, KG/cm^2.$$

We see that the condition $0.1 < \frac{\sqrt[3]{m}}{r} = \frac{\sqrt[3]{200}}{30} = 0.195 < 10$ is satisfied.

Note: The examples show that the Sadovsky's formula gives **overrating outcomes.**

Example 2.

Use the outcome of example 1, ($m = 200kg, r = 30m$) to find:

a. Time - interval τ (period of SW), $\tau = 0.00115\sqrt[6]{m} \cdot \sqrt[2]{r}$.
b. The specific hydrostatic impulse i, $i = \int_0^\tau \Delta P dt = 168\sqrt[3]{m^2}\,/r$.
c. Dynamic impulse, $j = \int_0^\tau \rho \cdot u^2 dt = 221 \cdot \sqrt[3]{m} \cdot (\frac{\sqrt[3]{m}}{r})^{2.5}$
d. Total impulse, $I = i + j$, impact force of the SW on the unit of area.

Solution:

Substituting in the above formulae we find:

a. Time – interval, $\tau = 0.00115\sqrt[6]{m} \cdot \sqrt[2]{r} = 0.00115\sqrt[6]{200} \cdot \sqrt[2]{30} = 0.015\ sec.$

b. Specific hydrostatic impulse, $i = 168\sqrt[3]{m^2}/r = 168\sqrt[3]{200^2}/30 = 191.52\ Pa \cdot s$

c. Dynamic impulse, $j = 221\sqrt[3]{m} \cdot (\frac{\sqrt[3]{m}}{r})^{2.5} = 221 \cdot \sqrt[3]{200} \cdot (\frac{\sqrt[3]{200}}{30})^{2.5} = 21.70\ Pa \cdot s$

d. Total Impulse, $I = i + j = 191.52 + 21.70 = 213.21\ Pa \cdot s$.

Example 3.
A reactive projectile explodes in air at a distance $r = 15\ m$ meters from a military jet. Estimate the maximum overpressure ΔP_H at the front of the shock wave created by explosion in air of a TNT equivalent explosive charge with mass $m = 50\ kg$.

Solution:
Substituting in formula (2.10), $m = 50\ kg$ and $r = 15\ m$, we find that

$$\Delta P_H = 1.007 \cdot (\frac{\sqrt[3]{50}}{15}) + 1.50 \cdot \left(\frac{\sqrt[3]{50}}{15}\right)^2 + 6.04 \cdot \left(\frac{\sqrt[3]{50}}{15}\right)^3 - 0.0196 = 0.408\ KG/cm^2.$$

Example 4.
Estimate the mass of TNT spherical charge that needed to explode in unconfined air in order that at a distance $r = 20\ m$ from the center of explosion the maximum overpressure will be $\Delta P_H = 0.35\ KG/cm^2$.

Solution:
First, we substitute in formula (2.6), $\Delta P_H = 0.35\ KG/cm^2$, we have:

$$0.35 = 0.79 \cdot \left(\frac{\sqrt[3]{m}}{r}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3$$

Solving the above third-degree equation with respect to $(\sqrt[3]{m}/r)$, (using a graphing calculator, or any other Math software), we obtain:

$$(\sqrt[3]{m}/r) = 0.215.$$

Hence, substituting $r = 20$, we find:

$$m = (0.215 \cdot r)^3 = (0.215 \cdot 20)^3 = 79.51\ kg\ \text{TNT}$$

Example 5.
Estimate the distance r from the center of a TNT spherical charge with mass $m = 1000\ kg$, that is needed to explode in air in order that the maximum overpressure should be $\Delta P_H = 0.50\ \ KG/cm^2$.

Solution:

First, we substitute in equation (2.6), $\Delta P_H = 0.50\ \ KG/cm^2$. We have:

$$0.50 = 0.79 \cdot \left(\frac{\sqrt[3]{m}}{r}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3$$

Solving above equation with respect to $(\sqrt[3]{m}/r)$ we find:

$$(\sqrt[3]{m}/r) = 0.263.$$

Hence

$$r = 3.64\sqrt[3]{m} = 3.64\sqrt[3]{1000} = 38\ m.$$

Example 6.
Find the peak overpressure at the distance $r = 11r_0$ from the center of a TNT explosive charge of mass m detonated in air.

Solution:
Substituting in (2.8b) we find:

$$\Delta P_H = 25000\left(\frac{r_0}{11r_0}\right)^3 = 18.78\ \ KG/cm^2$$

2.4 Lethal Range (Very large Explosions and Nuclear Explosions; 50% casualties)

For the personnel, the lethal range, due to large/nuclear air blast, is the distance from the center of explosion of a TNT charge, where the maximum overpressure is $\Delta P_H = 0.35\ \ KG/cm^2 \approx 5\ Psi$.

Let's estimate the lethal range for a mass m. Substituting $\Delta P_H = 0.35\ \ KG/cm^2$ in equation (2.6), we have

$$0.79 \cdot \left(\frac{\sqrt[3]{m}}{r}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3 = 0.35 \qquad (2.11)$$

Solving the above equation with respect to $(\sqrt[3]{m}/r)$, we have

$$(\sqrt[3]{m}/r) = 0.215. \qquad (2.12)$$

Hence, we find the radius of the lethal range

$$r = 4.66\sqrt[3]{m}. \qquad (2.13)$$

For example, the lethal range of an explosion obtained from the explosion of an atomic device of a TNT equivalent charge of $m = 1\ KT = 1{,}000{,}000\ kg$ is

$$r = 4.66\sqrt[3]{m} = 4.66\sqrt[3]{1{,}000{,}000} = 466\ m.$$

2.5 Shock Wave Scaling Rule

The solution (2.12), of equation (2.11) shows that for a given value of ΔP_H the ratio $(\sqrt[3]{m}/r)$ is a constant. Applying (2.12), for two different masses m_1, m_2 and their corresponding radiuses r_1, r_2, we can write

$$(\sqrt[3]{m_1}/r_1) = (\sqrt[3]{m_2}/r_2) \qquad (2.14)$$

The equation (2.14) is called the scaling rule.
Scaling rule is valid for the Ground Surface Explosion as well.
The scaling rule can be applied also for other parameters. For example, for the air-burst altitude h:

$$(\sqrt[3]{m_1}/h_1) = (\sqrt[3]{m_2}/h_2).$$

Example 1 (Application of Scaling Rule).

Let' assume that the pick overpressure of a SW, created by a TNT charge with mass $m = 1{,}000\ kg$, at the distance $r_1 = 46.60\ m$ is $\Delta P_H = 0.35\ KG/cm^2$. What is the distance r_2 from the TNT explosive charge where we expect to have the same pick overpressure if the exploding mass is $m_2 = 10{,}000\ kg$?

Solution:
Substituting in equation (2.14), we can write:

$$(\sqrt[3]{1000}/46.60) = (\sqrt[3]{10000}/r_2)$$

Hence, we find that $r_2 = 100\ m$.

2.6 Parameters at the Front of SW

At the front of SW in air, the characteristics parameters, speed of SW, pressure, velocity of air particles, density, temperature, speed of sound, change instantly from the respective values in the stationary air to the corresponding values at the front of SW.

Once we know the maximum overpressure at the front of SW, using the following equations we are able to find all the remaining parameters, i.e.

- Speed of SW, D:

$$D = 340(1 + 0.857\Delta P_H)^{\frac{1}{2}} \qquad (214a)$$

Speed of air particles, u_H:

$$u_H = 243 \cdot \frac{\Delta P_H}{(1+0.857\Delta P_H)^{1/2}} \qquad (214b)$$

- Density of air, ρ_H:

$$\rho_H = 1.26 \cdot \frac{6\Delta P_H+7}{\Delta P_H+7} \qquad (214c)$$

Temperature of air, T_H :

$$T_H = 288\frac{(7+\Delta P_H)(\Delta P_H+1)}{6\Delta P_H+7} \qquad (214d)$$

- Speed of sound, c_H :

$$c_H = 340 \cdot \frac{[(1+\Delta P_H)\cdot(7+\Delta P_H)]^{1/2}}{(6\Delta P_H+7)^{1/2}} \qquad (213e)$$

or

$$c_H = 20.05\sqrt{T_H} \qquad (214f)$$

Note that the above formulas are valid also when we have a SW, produced by a ground explosion, or we have a reflected SW.

Example 1.

The max overpressure of a SW is $\Delta P_H = 0.2\ KG/cm^2$. Find the parameters at the front of SW.

Solution:
Substituting, in the formulas 2.14a -2.14e, we find:

$$D = 368\, m/s,\ \ u_H = 45\, m/s,\ \rho_H = 1.435\, kg/m^3,\ T_H = 303.4\, °K,\ c_H = 349 m/s.$$

Example 2 (Arrival time of SW).
An explosive charge of mass $m = 100\ kg$ TNT detonates on air. Find the time t of arrival of the shock wave at a distance $r = 60\ m$ from the center of explosion.
Solution:
Since the velocity D of the front wave of SW changes with the maximum overpressure ΔP_H, and so with the distance r, we will consider an average constant velocity $\overline{D}$, that is equal to the velocity of SW at half of distance, i.e. $r/2 = 60/2 = 30\ m$.
Substituting in (2.6), we have:

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{100}}{30}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{100}}{30}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{100}}{30}\right)^3 = 0.207\ \ KG/cm^2.$$

Using (2.14a), we find the velocity of SW, at 30 m is

$$\overline{D} = 340(1 + 0.857 \cdot 0.207)^{1/2} = 369\ m/s.$$

Time of arrival of SW at $r = 60\ m$ is approximately

$$t = r/\overline{D} = 60/365.83 = 0.16\ sec\ .$$

Note: The estimated time is something greater that the real time.

3. Ground Surface Explosion

3.1 Maximum Overpressure of SW Produced by a Ground Explosion

The explosion of an explosive charge of mass m on surface of an absolutely rigid ground, or on any other hard surface, is accompanied by a shock wave, which expands along the hemisphere centered on the ground surface. Because the energy of the shock wave is distributed over half of the sphere, the parameters of the ground shock wave, at a distance r from the center of explosive charge, are the same as the parameters of air shock wave (at same distance r), but created by the explosion of a double mass ($2m$) of explosive charge.

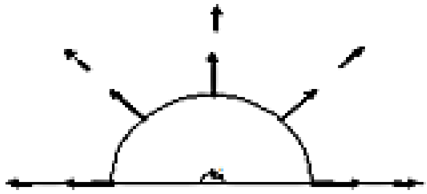


Fig. 5

Thus, the maximum overpressure in the front of the ground shock wave can be estimated substituting in equations (2.6), the mass $2m$ instead of m. For the maximum overpressure of the ground shock wave we obtain

$$\Delta P_H = 0.79 \cdot (\frac{\sqrt[3]{2m}}{r}) + 2.57 \cdot \left(\frac{\sqrt[3]{2m}}{r}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{2m}}{r}\right)^3, \tag{3.1}$$

valid for the pick overpressure that satisfies the condition

$$0.1\ KG/cm^2 \le \Delta P_H \le 10\ KG/cm^2, \tag{3.2}$$

and for any mass m of TNT charge.

Equation (3.1) can be written:

- $$\Delta P_H = 1.00 \cdot (\frac{\sqrt[3]{m}}{r}) + 4.08 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3, \tag{3.3}$$

  valid for $0.1\ KG/cm^2 \le \Delta P_H \le 10\ KG/cm^2$.

In a similar way, from equations (2.9), (2.10) we obtain:

- $$\Delta P_H = 13.84 \cdot m/r^3 + 1\ , \tag{3.4}$$

  valid for $\Delta P_H \ge 10\ KG/cm^2$.

- $$\Delta P_H = 1.27 \cdot (\frac{\sqrt[3]{m}}{r}) + 2.40 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 12.10 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3 - 0.0196 \tag{3.5}$$

  valid for $0.1\ KG/cm^2 \le \Delta P_H \le 10\ KG/cm^2$.

The equation (3.3), as well as formulae (3.4), (3.5), need corrections since, in general, the surface of the target (surface of the ground, façade of the wall, or water) is not absolutely rigid, but they can be deformed or damaged during the ground explosion. Part of the energy of explosion is used to deform, or damage the non-absolutely rigid surface. For that reason, in formulas (3.3), (3.4), (3.5) we introduce a non- rigidity factor β, which multiplies mass m. The values of β are given in Table 3 [4, p. 479].

In general, if the factor β is unknown, it can be considered equal to 0.80, i.e. $\beta = 0.80$ [6], [7].

Thus, for the max overpressure on the front of shock wave of a ground explosion we have the corrected formulae:

- $\Delta P_H = 1.00 \cdot (\frac{\sqrt[3]{\beta m}}{r}) + 4.08 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3,$ (3.6)
- $\Delta P_H = 13.84 \cdot \left(\beta \frac{m}{r^3}\right) + 1\,,$ (3.7)

  valid for $\Delta P_H \geq 10\ KG/cm^2$.
- $\Delta P_H = 1.27 \cdot (\frac{\sqrt[3]{\beta m}}{r}) + 2.40 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 21.10 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 - 0.0196$ (3.8)

Table 3.

Type of Ground Surface	Steel	Iron – Concrete	Concrete, Rocky ground	Dense loam, Clay	Average dense ground, vegetable soil	Water
β	1	0.95 - 1	0.85 – 0.9	0.7 – 0.8	0.6 – 0.65	0.55 – 0.6

For ground explosion, we can write:

The time-interval of positive phase, i.e., period τ (sec) of SW is given by the formula

$$\tau = 0.0013\sqrt[6]{m} \cdot \sqrt[2]{r}. \quad (3.9)$$

Since the SW of explosion is result of the detonation on a rigid ground of a mass of explosive charge 2 times greater than the explosive mass of the airburst, in (2.2), we introduced a factor (2) that multiplies the mass m in order to get (3.9). We did the same to get the impulses i and j using (2.8) and (2.8a) respectively.

If the surface is not an absolutely rigid one, we multiply the explosive mass the non-rigidity factor β in the respective formulas.

Thus, from (2.8), for the specific hydrostatic impulse of the positive phase of SW we can write:

$$i = \int_0^\tau \Delta P \cdot dt = 266.68\sqrt[3]{(\beta\ \mathrm{m})^2}\,/r. \quad (3.10)$$

(SI unit of impulse is $P_a \cdot s = N \cdot s/m^2$).

Another important quantity of SW is the dynamic impulse of the particles of air moving behind the front of SW (fig. 4). Using (2.8a), for the dynamic impulse we have:

$$j = \int_0^\tau \rho u^2 dt = 496.13 \cdot \sqrt[3]{\beta m} \cdot (\frac{\sqrt[3]{\beta m}}{r})^{2.5} \quad (3.11)$$

Keep in mind that for the absolutely rigid surface the non-rigidity factor $\beta = 1$.

Using equations (3.6) and (3.7), we can obtain the distance r, from the center of explosion of an explosive charge m, where it is observed a certain max overpressure ΔP_H.

When the SW impacts on an obstacle under an angle α, the dynamic impulse is

$$j_\alpha = j \cdot cos^2(\alpha), \quad (3.11a)$$

while the total impulse is

$$i_T = (i + j \cdot cos^2(\alpha)]. \quad (3.11b)$$

If the impact angle is $\alpha = 90°$, i.e. when SW travel parallel to the surface of the obstacle, then the dynamic impulse is zero, $j_{90} = 0$, while the total impulse is equal to the static one. Indeed, since the particles of air move parallel to the obstacle (ground, wall, etc.), there is no interaction with the obstacle, and so the dynamic pressure is zero.

That is the reason that during nuclear explosions the personnel must lie down to avoid, not only the deadly debris put in motion by the strong "wind", but also the risk of falling down, or knocking other objects.

Note that the non-rigidity factor β must be used when a SW impacts on an obstacle (wall, facade, hill, etc.) that is not perfectly rigid.

In formula (3.9), for time-interval, we have to introduce as well the non-rigidity factor β. Thus, we have:

$$\tau = 0.0013 \sqrt[6]{\beta m} \cdot \sqrt[2]{r}. \quad (3.9a)$$

Example 1.

Find the distance r from the center of ground explosion where max overpressure is $\Delta P_H = 0.35\ KG/cm^2$.

Solution:

Substituting $\Delta P_H = 0.35\ KG/cm^2$ in (3.6) and (3.8), we have respectively:

$$1.00 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 = 0.35\ , \quad (3.12)$$

and

$$1.27 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right) + 2.40 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 21.10 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 - 0.019 = 0.35 \tag{3.13}$$

Solving each equation for the unknown inside the parenthesis we get respectively

$$\left(\frac{\sqrt[3]{\beta m}}{r}\right) = 0.170 \quad \text{and} \quad \left(\frac{\sqrt[3]{\beta m}}{r}\right) = 0.165 \tag{3.14}$$

Hence, for the distance where is observed the overpressure $\Delta P_H = 0.35\ KG/cm^2$, we can write respectively:

$$r = 5.88 \cdot \sqrt[3]{\beta m}, \qquad r = 6.06 \cdot \sqrt[3]{\beta m}. \tag{3.15}$$

Example 2

Consider an explosive charge mass $m = 160\ kg$ that is detonated on a concrete flat surface, $\beta = 0.85$. Find:

- the distance from the center of ground explosion where max overpressure is $\Delta P_H = 0.35\ KG/cm^2$.
- The total impulse at that distance.

Solution:

(a) Substituting $m = 160\ kg$, $\beta = 0.85$ in both equations (3.15), we practically have the same outcome:

$$r = 5.88 \cdot \sqrt[3]{\beta m} = 5.88 \cdot \sqrt[3]{0.85 \cdot 160} = 30.24\ m.$$
$$r = 6.06 \cdot \sqrt[3]{\beta m} = 6.06 \cdot \sqrt[3]{0.85 \cdot 160} = 31.17\ m.$$

(b) Since the SW travels parallel to the ground surface, i.e. the angle α that the direction of air particles (“wind”) form with the ground is $\alpha = 90°$. So,

$$j_{\alpha(90)} = j \cdot cos^2(\alpha) = j \cdot cos^2(90°) = 0.$$

The total impulse is

$$i_T = i = 266.68 \cdot \sqrt[3]{(\beta\ \mathrm{m})^2}/r = 266.68 \cdot \sqrt[3]{(\ 0.85 \cdot 160)^2}/30 = \ 235.1\ Pa \cdot s.$$

The load exerted on the ground is due to the static impulse.

Example 3.

Consider an explosive charge mass $m = 1{,}000\ kg$ that is detonated on a flat clay ground, $\beta = 0.75$. Find the distance from the center of explosion where max overpressure is $\Delta P_H = 0.50\ KG/cm^2$.

Solution:
We need to find the distance r where max overpressure is $\Delta P_H = 0.50\ KG/cm^2$.

Substituting $\Delta P_H = 0.50\ KG/cm^2$ in (3.6) and (3.8), we have respectively:

$$1.00 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 = 0.50, \qquad (3.16)$$

and

$$1.27 \cdot (\frac{\sqrt[3]{\beta m}}{r}) + 2.40 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 21.10 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 - 0.019{=}0.50 \qquad (3.16a)$$

Solving each equation for the unknown inside the parenthesis we get respectively

$$\left(\frac{\sqrt[3]{\beta m}}{r}\right) = 0.21 \quad \text{and} \quad \left(\frac{\sqrt[3]{\beta m}}{r}\right) = 0.20 \qquad (3.17)$$

Hence, for the distance where it is observed the overpressure $\Delta P_H = 0.50\ KG/cm^2$, we can write respectively:

$$r = 4.76 \cdot \sqrt[3]{\beta m}, \qquad r = 5 \cdot \sqrt[3]{\beta m}.$$

Substituting $m = 1000$, and $\beta = 0.75$, in the above equations we find the required distance, respectively

$$r = 4.76 \cdot \sqrt[3]{\beta m} = 4.76 \cdot \sqrt[3]{0.75 \cdot 1000} = 43.25\ m$$

and

$$r = 5 \cdot \sqrt[3]{\beta m} = 4.48 \cdot \sqrt[3]{0.75 \cdot 1000} = 45.43\ m.$$

Example 4 (Clearing a Mine-Field).
To clear an antitank mine-field using conventional explosive devices it is needed a peak overpressure $\Delta P_H = 20\ KG/cm^2$.
Find the mass of an explosive TNT charge that is needed to clear the mines at a distance $r = 2\ m$ from the center of explosion, if the field is a rocky-ground, $\beta = 0.85$.

Solution:

Since the peak overpressure is $\Delta P_H = 20\, KG/cm^2 > 10\, KG/cm^2$, we have to use equation (3.7), i.e.

- $\Delta P_H = 13.4 \cdot \left(\beta \frac{m}{r^3}\right) + 1$.

Substituting we can write:

$$20 = 13.4 \cdot \left(0.85 \frac{m}{2^3}\right) + 1.$$

Solving the above equation, we find that mass of the TNT charge should be not smaller than $m = 13.35\, kg$.

Note: Solving the above equation for the mass m, we find the TNT mass of the explosive charge that is needed to clear the obstacles inside a circle with radius r.

- $m = 0.0749 r^3 (\Delta P_H - 1)/\beta$.

Example 5.
Use the data of Example 4, $m = 13.35\, kg\, TNT$, $r = 2\, m$, $\beta = 0.85$ to find:
the time-interval, τ, the static impulse i and the dynamic impulse j that correspond to $\Delta P_H = 20\, KG/cm^2$.

Solution:
Time interval, τ

a. $\tau = 0.0013 \sqrt[6]{\beta m} \cdot \sqrt[2]{r} = 0.0013 \cdot \sqrt[6]{0.85 \cdot 13.35} \cdot \sqrt{2} = 0.0028$ sec.

Static impulse i,

$b.\ i = \int_0^\tau \Delta P dt = 266.70 \sqrt[3]{(\beta m)^2}\, /r = 266.70 \sqrt[3]{(0.85 \cdot 13.35)^2}/2 = 673.38\, Pa \cdot sec.$,

Dynamic impulse:

$$c.\ j = \int_0^\tau \rho \cdot u^2 dt = 496.13 \cdot \sqrt[3]{0.85 \cdot 13.35} \cdot \left(\frac{\sqrt[3]{0.85 \cdot 13.35}}{2}\right)^{2.5} = 1491.9.36\, Pa \cdot sec\ .$$

Note that the dynamic impulse does not interact with the mine, since the particles of air of SW are parallel to the ground surface.

Example 6 (Arrival time of SW).

The largest conventional bomb Blu-82/B has approximately an equivalent TNT charge of mass $m = 12{,}300\ kg$.

a. Find the arrival time t of SW at a distance $r =\ 220$ from the center of explosion, if it is detonated on a rigid ground, $\beta = 1$.

b. Does a person have enough time, since the instant he saw the detonation light, to lie down on his/her belly, heels on the direction of center of explosion (as it is recommended for nuclear bombs)?

Solution:

The average constant velocity $\overline{D}$, that is equal to the velocity of SW at half of the distance, i.e. $220/2 = 110\ m$.

a. Substituting in (3.3), we have:

$$\Delta P_H = 1.00 \cdot \left(\frac{\sqrt[3]{12{,}300}}{110}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{12{,}300}}{110}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{12{,}300}}{110}\right)^3 = 0.505\ KG/cm^2 .$$

Using (2.14a), we find the velocity of SW, at $r = 110\ m$ is

$$\overline{D} = 340(1 + 0.857 \cdot 0.505)^{1/2} = 407\ m/sec.$$

Time of arrival of SW at $r = 220\ m$ is approximately

$$t = r/\overline{D} = 220/407 = 0.54\ sec .$$

b. No.

3.2 Long Explosive Charge, Bangalore Torpedo

Long explosive charges (LEC) are mainly used to clear mine-fields to open free pathways for personnel or vehicles (tanks, artillery etc.). The SW created by a long explosive charge (fig.6) that explodes on the ground, is a half-cylindrical surface with axis the LEC itself.

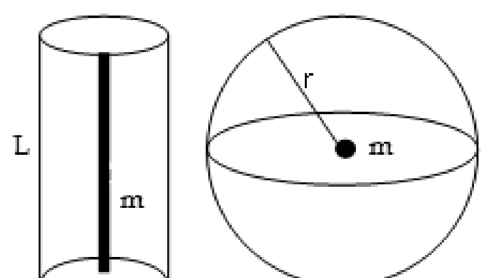


Fig. 6

Assume that a spherical explosive charge (SEC) and a **long explosive charge (LEC**), are ground explosions. The peak overpressure at the front of SW created by LEC can be estimated using the equations obtained for SEC detonated on the ground (3.6, 3.7, 3.8). For that, let's consider as an example, the equation (3.7), valid for a ground explosion of a TNT mass m, radius r, i.e.

- $\Delta P_H = 13.4 \cdot \left(\beta \frac{m}{r^3}\right) + 1$, (3.15)

where ($\Delta P_H \geq 10\ KG/cm^2$.) while r is the distance from the center of explosive where is observed the maximum overpressure ΔP_H.

We require that the max overpressure ΔP_H, at a distance r from the center of the spherical TNT charge (mass m_1) to be equal to max overpressure ΔP_H created by the long TNT charge (mass m). TNT of LEC is uniformly distributed along the axis with length L. (The linear density of the explosive is $\rho_L = m/L$). The SW front of the long explosive charge, at the distance r , is a half-cylinder surface with radius of the base r and length L, i.e. $S = \pi \cdot rL$. The SW front of the spherical charge has a hemispheric form with radius r, i.e. $S_1 = 2\pi r^2$.

The max overpressure ΔP_H, as a function of the energy of SW, would be the same for both charges if the flux of energy E at the unit of surface S will be equal to the flux of energy E_1 at the unit of surface S_1. Thus, we can write:

$$E_1/(\pi rL) = E/(2\pi r^2) \quad (3.16)$$

Since the energy is proportional to the mass of the explosive charge, we can write

$$m/(\pi rL) = m_1/(2\pi r^2). \quad (3.17)$$

Hence,

$$m_1 = 2r(m/L). \quad (3.18)$$

Substituting m_1 formally for m, in (3.15) we obtain the max overpressure of the SW of the long explosive charge:

$$\Delta P_H = 26.8 \cdot \beta \left(\frac{m}{L}\right)/r^2 + 1, \quad (3.19)$$

which is valid for ($\Delta P_H \geq 10\ KG/cm^2$). Note that $\rho_L = m/L$ is the linear density of the long explosive charge.

Note. For max overpressure $0.1\ KG/cm^2 \leq \Delta P_H \leq 10\ KG/cm^2$, the equation of maximum overpressure of SW is obtain by formally substituting in equation (3.6) and (3.8), instead of m, i.e. $m_1 = 2mr/L$, (See example 2 below).

Example 1.
The Russian military handbook [8, p.221], shows that to detonate any type of mine on a mine-field, the maximum overpressure of SW of the explosion must be not less than 20 KG/cm^2 , i.e. $\Delta P_H \geq 20\ KG/cm^2$.

Find the linear density, $\rho_L = m/L$ of the TNT needed to construct a LEC that would be necessary to open a mine-free pathway with width $2r = 3.6\ m$. The ground is a clay soil, with $\beta = 0.80$.

Solution:
Solving equation (3.19, with respect to (m/L), we can write:

$$m/L = (\Delta P_H - 1)r^2/(26.8\,\beta). \tag{3.20}$$

Substituting we find that the linear density is

$$m/L = (20 - 1) \cdot (1.8)^2/(26.8\ \cdot 0.80) = 2.87\ kg/\ m.$$

Example 2.
Convert the spherical SW max overpressure equation (3.6) into the equation of max overpressure of SW generated by a long explosive cylindrical charge using equation (3.6), i.e.

$$\Delta P_H = 1.00 \cdot (\frac{\sqrt[3]{\beta m}}{r}) + 4.08 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3.$$

Use the obtained long explosive charge equation to find the linear density of TNT explosive charge $\rho_L = m/L$, if we need to clear a pathway in a mine-field with a width of $2r = 4\ meter$. The long explosive charge is placed on a rocky ground with correction factor $\beta = 0.85$, while the max overpressure is $\Delta P_H = 8\ KG/cm^2$.

Solution:
Substituting m_1, $m_1 = \ 2r(m/L)$, given by (3.18), instead of m in the above equation we find the equation:

$$\Delta P_H = 1.00 \cdot (\sqrt[3]{\frac{2\beta m}{Lr^2}}) + 4.08 \cdot \left(\sqrt[3]{\frac{2\beta m}{Lr^2}}\right)^2 + 12.56 \cdot \left(\sqrt[3]{\frac{2\beta m}{Lr^2}}\right)^3 \tag{3.21}$$

where $0.1\ KG/cm^2 \leq \Delta P_H \leq 10\ KG/cm^2$.

Since, $0.1\ KG/cm^2 < \Delta P_H = 8 < 10\ KG/cm^2$, we will use equation (3.21).

Substituting $\Delta P_H = 8$ in (3.21) we can write:

$$\Delta P_H = 1.00 \cdot \left(\sqrt[3]{\frac{2\beta m}{Lr^2}}\right) + 4.08 \cdot \left(\sqrt[3]{\frac{2\beta m}{Lr^2}}\right)^2 + 12.56 \cdot \left(\sqrt[3]{\frac{2\beta m}{Lr^2}}\right)^3 = 8$$

Solving for the unknown inside the parentheses we find

$$\sqrt[3]{\frac{2\beta m}{Lr^2}} = 0.738. \tag{3.22}$$

Substituting $r = 2$, $\beta = 0.85$ and solving for linear density, $\rho_L = m/L$, we find that the linear density is

$$\rho_L = \frac{m}{L} = 0.738^3 \cdot \frac{r^2}{2\beta} = 0.738^3 \cdot \frac{2^2}{2 \cdot 0.85} = 0.945 \frac{kg}{m}.$$

Note. To open a pathway $L = 10\ meter$ long we need a quantity of

$$L = 0.985 \cdot 10 = 9.45\ kg \text{ TNT}.$$

Example 4. (Bangalore Torpedo)

A standard Bangalore Torpedo, with linear density $\rho_L = m/L = 1.6\ kg/m$, detonated on a ground with anti-personnel mines, clears of a pathway of $2r = 7\ meter$. Find the max overpressure ΔP_H needed to clear the pathway from anti-personnel mines.

Solution:

Since we do not know the max overpressure ΔP_H, we are going to use (3.19) and (3.20). Assume that the ground is a clay soil, $\beta = 0.75$. Substituting in (3.19) we obtain:

$$\Delta P_H = 26.8 \cdot \frac{\beta\left(\frac{m}{L}\right)}{r^2} + 1 = 26.8 \cdot \frac{0.75(1.6)}{3.5^2} + 1 = 3.625\ KG/cm^2.$$

The outcome obtained using (3.19) is not the right one since it is incompatible with the condition $(\Delta P_H \geq 10\ KG/cm^2)$. Thus, we need to use the equation (3.21).

Substituting in (3.21) we find that

$$\Delta P_H = 1.00 \cdot \left(\sqrt[3]{\frac{2 \cdot 0.75 \cdot 1.6}{3.5^2}}\right) + 4.08 \cdot \left(\sqrt[3]{\frac{2 \cdot 0.75 \cdot 1.6}{3.5^2}}\right)^2 + 12.56 \cdot \left(\sqrt[3]{\frac{2 \cdot 0.75 \cdot 1.6}{3.5^2}}\right)^3 = 4.43\ KG/cm^2.$$

Example 5.
Find the parameters at the front of a SW produced by a ground explosion of mass $m = 100\ kg\ TNT$, at a distance $r = 10\ m$. The explosion is on an absolutely rigid flat surface, $\beta = 1$.

Solution:
Substituting in equation (3.6), we have:

$$\Delta P_H = 1.00 \cdot \left(\frac{\sqrt[3]{100}}{10}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{100}}{10}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{100}}{10}\right)^3 = 0.454\, \frac{KG}{cm^2}.$$

Substituting in equations (2.14a) – (2.14e) we find:

$D = 402\ m/s,\ \ u_H = 95.4\ m/s$ (speed of wind), $\rho_H = 1.7\ kg/m^3,\ T_H = 322\ °K,\ c_H = 368\ m/s.$

4. Reflected Shock Wave

4.1 Reflection of Vertical SW

When a shock wave runs vertically into a flat obstacle (wall, flat ground, façade of a building, etc.), it is reflected from the surface, which is the border between the air and the obstacle. At that time, there is generated the reflected SW, (SWr), that propagates in air, and another SW that propagates into the obstacle material. The last one is result of the compression of the material of obstacle initiated by incident SW. SWr moves in the opposite direction of the vertical incident SW.
For example, the SWr is initiated during the explosion in air (of an explosive charge), when the SW impacts vertically on the ground surface, i.e. at the epicenter of explosion.

The max overpressure at the front of vertically reflected SWr, can be estimated by the equation [1], [4]:

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7}\,, \tag{4.1}$$

where ΔP_H is the max overpressure of the incident SW on the rigid surface.
Note that:

- When ΔP_H is very small ($\Delta P_H \approx 0$), from (4.1), we find that $\Delta P_r \approx 2\Delta P_H$.
- When ΔP_H is very large ($\Delta P_H \gg 7$), from (4.2), we find that $\Delta P_r \approx 8\Delta P_H$.

So, for the reflected SWr, the peak overpressure at the front of SWr can be 2 times to 8 times greater than the peak overpressure of the incident SW, i.e.

$$2\Delta P_H \leq \Delta P_r \leq 8\Delta P_H. \tag{4.2}$$

The increase of the peak overpressure, and in general, the increase of the overpressure at the positive phase of reflected SW, is due to the deceleration of the flux of air-particles (wind) by the obstacle.

Note. The time-interval and the total impulse can be estimated using the equations (2.7), (2.8) and (2.8a). The total impulse is:

$$I_r \approx 2(i+j) \tag{4.2a}$$

where

$$i = \int_0^\tau \Delta P \cdot dt\ , \qquad j = \int_0^\tau \rho u^2 dt \tag{4.2b}$$

are the specific impulses of incident SW.

Example 1.

An explosive charge of $m = 10\ kg\ TNT$ is detonated $h = 2.5\ m$ over the rocky ground ($\beta = 0.75$). Find the peak overpressure of the incident SW and the peak overpressure of the reflected SW at the epicenter of explosion if the ground is composed by clay. Find as well the total impulse.

Solution:

a. Substituting in (2.6), we find the peak overpressure of the incident SW is

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{0.75 \cdot 10}}{2.5}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{0.75 \cdot 10}}{2.5}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{0.75 \cdot 10}}{2.5}\right)^3 = 2.8\frac{KG}{cm^2}\ .$$

Using (4.1), we have:

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} = 2 \cdot 2.8 + 6 \cdot \frac{2.8^2}{2.8 + 7} = 10.4\frac{KG}{cm^2}\ .$$

Note. We see that the reflected peak overpressure at the epicenter of explosion is 3.71 times greater than the peak overpressure of the incident SW.

b. $i = \frac{168\sqrt[3]{m^2}}{r} = 312\ Pa \cdot sec,\quad j = 221 \cdot \sqrt[3]{m} \cdot (\frac{\sqrt[3]{m}}{r})^{2.5} = 328.25\ Pa \cdot sec$.

$I_r \approx 2(i + j) = 1280.5\ Pa \cdot sec$.

Example 2.
A person is standing next to the wall of a building, when a SW with peak overpressure $\Delta P_H = 0.35\ KG/cm^2$ run into the wall.
What is the peak overpressure of the reflected SW that is exerted on that person?

Solution:
Substituting in (4.1), we have

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} = 2 \cdot 0.35 + 6 \cdot \frac{0.35^2}{0.35+7} = 0.80\frac{KG}{cm^2}.$$

Note that the standing human being is subject to the incident SW and the reflected SWr.

Example 3.
A man standing next to a brick wall is hit by a SW generated by an explosive charge. Find the mass m of the explosive charge needed to be detonated on the ground at a standoff distance of $r = 10\ m$, from the wall if we want to have a reflected peak overpressure of $\Delta P_r = 1.00\ KG/cm^2$.

Solution:
Since we do not know the non-rigidity factor β that corresponds to the energy of SW lost during the reflection we can consider it $\beta = 0.80$.
Substituting $\Delta P_r = 1\ KG/cm^2$in (4.1), we can write:

$$2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} = 1.00$$

Solving the above equations for ΔP_H, we find: $\Delta P_H = 0.43\ KG/cm^2$.

Substituting $\Delta P_H = 0.43$, in (3.6), we have:

$$1.00 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 = 0.43.$$

Solving for $\sqrt[3]{\beta m}/r$, we can write $\sqrt[3]{\beta m}/r$=0.19.

Substituting, we have:

$$\sqrt[3]{0.80m}/10 = 0.19.$$

Hence, we find:

$$m = 8.57\ kg\ \text{TNT}.$$

4.2 Regular and Irregular Reflection of Shock Wave. Mach wave

The shock wave of an airburst of a given explosive charge m, detonated at an altitude h over the ground, is reflected when it reaches at any point on the ground surface (oblique reflection). As result, there are generated two other shock waves (fig. 4):

- the reflected shock wave, SWr that moves in opposite direction of incident wave. The spread of the SWr can be considered as if the center of the SWr is mirror reflection of the airburst of mass m.
- the so called "Mach wave", that travels parallel to the ground surface. Mach wave degenerates to a sound wave far away from the epicenter of explosion.

There is created also another SW that travels inside the ground (obstacle).

The region from the epicenter of explosion to the point C, where the Mach wave is created, is called the region of regular reflection. In this region, the angle of reflected SW and the angle of incident SW are equal. In this region, the angle α of the incident shock wave is less than a boundary value α_c, i.e. $\alpha \leq \alpha_c$.
Note that the angle α, is the angle between the incident SW and the vertical line CN (fig. 7), while α_c is the angle between mC and CN.

Incident angle α can be calculated using the formulas:

$$\tan(\alpha) = r/h, \qquad \sin(\alpha) = r/R, \quad \text{where} \quad R^2 = r^2 + h^2, \qquad (4.3)$$

where r is the distance from the epicenter of airburst, h is the altitude of explosive charge, R is the distance of the point of ground impact from the explosive charge.

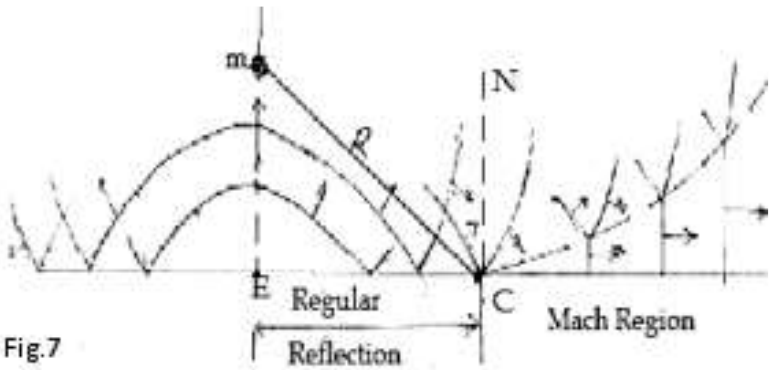


Fig.7

In the irregular region, beyond the point C (called critical point), there is the Mach wave, which propagates along the flat ground, while the triple point moves away from the ground. At the irregular region, the angle α of the incident shock wave is greater than boundary value α_c ($\alpha \geq \alpha_c$).

The point on the ground C that separates the regular reflection region from the Mach wave region is at the same time the point where the incident shock wave, the reflected shock wave and the Mach wave intersect.

At the triple point C, the incident angle α is equal to α_c, $\alpha = \alpha_c$). The triple point corresponds to the critical angle α_c.

4.3 Parameters of the Reflected SW and Mach Wave

In general, regular reflection and the Mach wave are observed when the incident SW runs obliquely into a flat surface of an obstacle (flat ground, hillside, wall of a building). Let's consider a SW that runs into a concrete wall (along line EC). fig. 7.

1. Assume that the incident SW falls on the ground, or on an obstacle (wall, hillside, facade, etc.) at an angle $\alpha \leq \alpha_c$ (regular reflection). The incident SW is reflected from the ground, or from an obstacle at an angle that, in general, is different from the incident angle α.

The maximum overpressure of the reflected SWr (in regular reflection region), can be estimated approximately using the formula [4]:

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} \cdot cos^2(\alpha) \qquad (4.4)$$

where ΔP_H is the peak overpressure of the incident SW.

The total impulse is:

$$I_r = 2[(i + j \cdot cos^2(\alpha)] \qquad (4.5)$$

where

$$i = \int_0^\tau \Delta P \cdot dt\, r\,, \quad j = \int_0^\tau \rho u^2 dt$$

are respectively the static and dynamic impulses of the incident SW.

For a given peak overpressure ΔP_H, in the front of the incident shock wave, the critical angle, α_c can be calculated using the equation:

$$\alpha_c = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot \Delta P_H)}\right) \tag{4.6}$$

In table 8 are presented some critical angles obtained using equation (4.6).

Table 8.

$\Delta P_H(KG/cm^2$	0	0.10	0.25	0.40	0.65	1.50
α_c (degree)	90	63	54	49	45	40

$\Delta P_H(KG/cm^2)$	2.00	4.00	10	20	50	100
α_c (degree)	39.4	39	39	39	39	39

Table 8 shows that for $\Delta P_H \geq 4$, the boundary value is almost constant and equal to approximately $\alpha_c = 39°$.

Note that for incident angle $\alpha = 0$, the formulas (4.4), (4.5) describe the parameters of the vertical incident SW (section 4.1).

2. If the incident SW forms an angle ($\alpha > \alpha_c$), (irregular reflection, Mach SW), then the max overpressure, at the front of Mach shock wave, is

$$\Delta P_r = \Delta P_H[1 + \frac{cos\alpha}{\cos(\alpha_c)} + \frac{6\Delta P_H}{\Delta P_H + 7} \cdot cos^2(\alpha)]. \tag{4.7}$$

The total impulse of the reflected SW is

$$I_r = i\left(1 + \frac{\cos\alpha}{\cos(\alpha_c)}\right) + 2j \cdot cos^2(\alpha), \tag{4.8}$$

where

$$i = \int_0^\tau \Delta P \cdot dt\, r\,, \quad j = \int_0^\tau \rho u^2 dt$$

are respectively the static and dynamic impulses of the incident SW.

Note: To estimate the scale of damage/injury, we have to consider the parameters of the reflected SWr, since the reflected max overpressure of SWr is always 2 to 8 times greater than the corresponding max overpressure of incident SW.

Example 1.

A TNT explosive charge of mass $m = 120\ kg$ explodes in air at an altitude of $h = 8\ m$. Find:

- max overpressure ΔP_r of the reflected SW at the epicenter of the airburst.
- max overpressure ΔP_r at the front of reflected SWr at the distance $r = 8\ m$ from the epicenter E of the airburst.
- max overpressure ΔP_r at the distance $r = 5\ m$ from the epicenter E.
- max overpressure at the front of SW, at a distance of $r = 5\ m$ if the same explosive charge will be detonated at the epicenter E of airburst.

Consider that the ground surface if absolutely rigid.

Solution:

1. Given: $m = 120\ kg$, h= $8\ m$, r= $0\ m$.

Substituting in (2.6), $m = 120$, $h = 8$ we find the max overpressure of incident SW at the epicenter E of airburst,

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{120}}{8}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{120}}{8}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{120}}{8}\right)^3 = 2.94\ KG/cm^2.$$

Using (4.1), we find the maximum overpressure of reflected SWr at the epicenter, (E):

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} = 2(2.94) + 6 \cdot (\frac{2.94)^2}{2.94+7} = 11.08\ KG/cm^2 .$$

2. Given: $m = 120\ kg$, $h = 8\ m$, r = $6\ m$.

Let's find the max overpressure of the incident SW at the point that is at the distance $r = 8\ m$ from the epicenter.

The incident angle of the SW, at the point located $r = 6\ m$ from the epicenter (E), is

$$\alpha = arctan(r/h) = \arctan(6/8) = 36.87° .$$

For the slant range R, we can write:

$$R^2 = r^2 + h^2 = 6^2 + 8^2 = 100.$$

Hence, we find $R = 10\ m$.

Substituting in (2.6), we get the max overpressure of incident SW at the slant distance $R = 10\ m$:

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{120}}{10}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{120}}{10}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{120}}{10}\right)^3 = 1.77\ KG/cm^2.$$

The critical angle is

$\alpha_c = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot \Delta P_H)}\right) = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot 1.77)}\right) =$ $39.6°$.

Since the incident angle α is less than the critical angle α_c, $\alpha = 36.87° < \alpha_c = 39.6°$, the given point is located at the irregular reflection region.

Substituting in (4.4) we find that the reflected max overpressure is:

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} \cdot cos^2(\alpha) = 2(1.77) + \frac{6(1.77)^2}{1.77 + 7} = 4.91 KG/cm^2$$

3. Given: $m = 120\ kg$, $h = 8\ m$, $\mathrm{r} = 8\ m$.

The incident angle of the SW, at the point located $r = 8\ m$ from the epicenter (E), is

$$\alpha = arctan(r/h) = \arctan(8/8) = 45°\ .$$

For the slant range R, we can write:

$$R^2 = r^2 + h^2 = 8^2 + 8^2 = 128.$$

Hence, we find $R = 11.31\ m$.

Substituting in (2.6), we get the max overpressure of incident SW at the slant distance $R = 11.31\ m$:

$$\Delta P_H = 0.79 \cdot \left(\frac{\sqrt[3]{120}}{11.31}\right) + 2.57 \cdot \left(\frac{\sqrt[3]{120}}{11.31}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{120}}{11.31}\right)^3 = 1.35\ KG/cm^2.$$

The critical angle is

$\alpha_c = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot \Delta P_H)}\right) = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot 1.35)}\right) =$ $40.25°$.

Since the incident angle α is greater than the critical angle α_c, $\alpha = 45° > \alpha_c = 40.25°$, the given point (r =8, h = 0) is located at the irregular reflection region.
Using (4.7), we can write:

$$\Delta P_r = \Delta P_H \left[1 + \frac{cos\alpha}{\cos(\alpha_c)} + \frac{6\Delta P_H}{\Delta P_H + 7} \cdot cos^2(\alpha)\right] = 1.35(1 + \frac{cos45}{\cos(40.25)} + \frac{6(1.35)}{1.35+7} \cdot cos^2(45) = 3.26KG/cm^2.$$

Example 2 (ref. [6])
In this example we are estimating the same quantities as in [6]:
A TNT explosive charge of mass $m = 100\ kg$ explodes on the ground at a standoff distance $r = 15\ m$ from a building wall. Find:

- max overpressure ΔP_r of the reflected SW at the standoff distance $r = 15\ m$ from the center of ground burst. Find as well the corresponding impulse, and time interval.
- max overpressure ΔP_r at the front of reflected SWr at the point of the wall that is $h = 12\ m$ over the ground. Find as well the corresponding impulse and the time-interval.

Consider that the ground surface and the surface of the wall are absolutely rigid.

Solution:

1. Given: $m = 100\ kg$, r= $15\ m$, angle of incident SW $\alpha = 0°$, vertical reflection.

Substituting in (2.6), $m = 100$, $r = 15$ we find the max overpressure of incident SW at the bottom of the wall:

$$\Delta P_H = 1.00 \cdot \left(\frac{\sqrt[3]{100}}{15}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{100}}{15}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{100}}{15}\right)^3 = 1.072\ KG/cm^2.$$

Using (4.1), we find the maximum overpressure of reflected SWr at the bottom of the wall:

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} = 2(1.072) + 6 \cdot (1.072)^2/(1.072 + 7) = 3\ KG/cm^2.$$

The static impulse of the incident SW is (see 3.10):

$$i = 266.68\sqrt[3]{(\mathrm{m})^2}/r = 266.68\sqrt[3]{(100)^2}/15 = 383.03\ Pa \cdot s\ .$$

The dynamic impulse of the particles of air moving behind the front of SW is (see 3.11):

$$j = 496.13 \cdot \sqrt[3]{m} \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^{2.5} = 496.13 \cdot \sqrt[3]{100} \cdot \left(\frac{\sqrt[3]{100}}{15}\right)^{2.5} = 122.66\ Pa \cdot s.$$

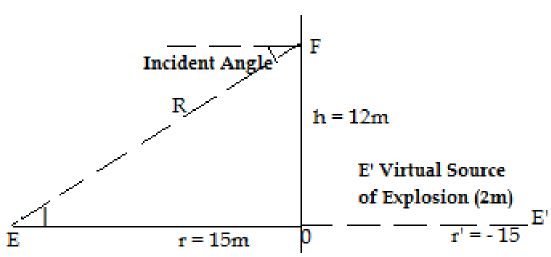
Total impulse is

$$I_r = 2[(i + j \cdot cos^2(\alpha)] = 2[(383.03) + (122.66) \cdot cos^2(0)] = 1011.4\ Pa \cdot s.$$

Time-interval is

$$\tau = 0.0013\sqrt[6]{2m} \cdot \sqrt[2]{r} = 0.0013\sqrt[6]{2 \cdot 100} \cdot \sqrt[2]{15} = 0.012\ sec.$$

In the above formulas we introduced the factor (2), since the SW is reflected from a rigid wall. We consider that the reflected SW is result of the SW created by a virtual TNT charge of the double mass detonated at the opposite side of the wall, at the distance r= $-15\ m$.



Note. Comparing the above obtained values with the corresponding values predicted by Remennikov [6]:
$\Delta P_r = 2.8KG/cm^2, I_r = 954\ Pa\ s,$
one can see that they are approximate. The predicted time interval, $\tau = 0.012\ sec,$ is different from the value $\tau = 0.0172\ sec$ predicted by Remennikov.

2. Given: $m = 100\ kg$, $h = 12\ m$, r= $15\ m$.

The incident angle of SW, at the point located $h = 12\ m$ from the bottom of the building is

$$\alpha = arctan(h/r) = \arctan(12/\ 15) = 39.81°.$$

Let's find the max overpressure of the incident SW at the given point on the wall. For the slant range R, we can write:

$$R^2 = r^2 + h^2 = 15^2 + 12^2 = 369\ m.$$

Hence, we find the slant range: $R = 19.21\ m$.

Substituting in (2.6), we get the max overpressure of the incident SW at the slant distance $R = 19.21\ m$:

$$\Delta P_H = 1.00 \cdot \left(\frac{\sqrt[3]{100}}{19.21}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{100}}{19.21}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{100}}{19.21}\right)^3 = 0.657\ KG/cm^2.$$

The critical angle that corresponds to max overpressure $\Delta P_H = 0.657\ KG/cm^2$ of the incident SW, is

$\alpha_c = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot \Delta P_H)}\right) = arcos\left(0.775\sqrt[3]{1 - \exp(-2.3 \cdot 0.657)}\right) =$ 44.50°.

Since the incident angle α is smaller than the critical angle α_c, i.e.

$\alpha\ = 39.81 < \alpha_c = 44.50°$, the given point is located at the regular reflection region.

Substituting in (4.4) we find that the max overpressure of the reflection SWr is:

$$\Delta P_r = 2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} \cdot cos^2(\alpha) = 2(0.657) + \frac{6(0.657)^2}{0.657+7} \cdot cos^2(39.81) = 1.65\ KG/cm^2\ ,$$

or

$\Delta P_r = 161.8\ kPa = 23.47\ psi.$ (Remennikov 146 kPa)

Note that the max overpressure at the wall decreases continuously from $\Delta P_r = 3\ KG/cm^2$(at the bottom of the wall) to $\Delta P_r = 1.65\ KG/cm^2$(at the point located on the wall at the height $h = 12\ m$).

Estimation of Impulse: Given $m = 100\ kg$, $h = 12\ m$, r= $15\ m$, $\alpha\ = 39.81°$.

The static impulse of the incident SW is (see 3.10):

$$i = 266.68\sqrt[3]{(\mathrm{m})^2}/R = 266.68\sqrt[3]{(100)^2}/19.21 = 299.09\ Pa \cdot s\ .$$

The dynamic impulse of the particles of air moving behind the front of SW is (see 3.11):

$$j = 496.13 \cdot \sqrt[3]{m} \cdot (\frac{\sqrt[3]{m}}{R})^{2.5} = 496.13 \cdot \sqrt[3]{100} \cdot (\frac{\sqrt[3]{100}}{19.21})^{2.5} = 66.09\ Pa \cdot s.$$

Total impulse is

$$I_r = 2[(i + j \cdot cos^2(\alpha)] = 2[(299.09) + (66.09) \cdot cos^2(39.81)] = 676.17\ Pa \cdot s.$$

Remennikov Prediction: 715 kPa s

Time-interval is

$$\tau = 0.0013\sqrt[6]{2m} \cdot \sqrt[2]{R} = 0.0013\sqrt[6]{2 \cdot 100} \cdot \sqrt[2]{19.21} = 0.014\ sec.$$

(Remennikov: 0.019 sec)

5. Injury from Shock Wave

5.1 Injury or Damage Criteria

Shock Wave of an explosion, hitting the human body, can cause different traumas and injuries, even deadly. The injury/trauma is result of:

- Primary action of SW, or and action of SWr, (the overpressure of positive phase).
- Secondary action of SW:
  1. The impact on human body of small fragments (debris, etc.) that are set into motion by SW, or are formed from shattered metallic body of the bomb,
  2. Damage/destruction of the buildings, shelters, etc. where the people are trapped inside.
  3. Fall of the person on the ground, or other objects, as result of the powerful "wind" that accompany the blast of large explosions.

Primary Action of SW.

The evaluation of the level of injury (damage), resulting from the interaction of SW of an explosion with a human being (obstacle) exposed to it, can be described using the peak overpressure ΔP_H, specific impulse i of the compressed phase, and the time-interval τ of the positive phase, which depend on the mass of the explosive charge and the distance from the center of explosion.

- If the length L of the SW is much greater than the characteristic dimension of a person (object) then the interaction is due to overpressure of SW, and can be estimated using as indicator the peak overpressure ΔP_H.
  Approximately, the length of SW can be estimated by the formula $L = c_0 \cdot \tau$, where c_0 is the standard speed of sound in air. As a matter of fact, the length of SW is determined by the formula $L = D \cdot \tau$ where D is the supersonic speed of SW which decreases as the SW travels away (see formula 2.14a).

- The peak overpressure estimation of the level of injury (destruction) is done also for very large explosive charges, if $\tau \geq 10T$, where T is the period of self-oscillation of the object that is given in many handbooks.

- The impulse estimation of the level of injury (damage) is done when $\tau < 0.25\,T$ (small explosive charges).

- For, 0.25 T< $\tau < 10\,T$ the estimation is performed using both the peak overpressure and impulse.

Large explosions have large time-intervals of the positive phase, τ, as well as large lengths of SW, while explosion of small conventional explosives have small time-interval and length.

It is known that for a small explosion, a given level of injury (damage) will occur at a higher max overpressure ΔP_{Hsmall}, than the smaller peak overpressure ΔP_{HLarge} of a large explosion (which has a longer time interval τ of the positive phase duration), i.e.

$$\Delta P_{HSmall)} > \Delta P_{H(Large)}$$

if

$$\tau(small) < \tau(Large).$$

See illustration examples (1) and (2) below.

Note that the level of injury, estimated using as criteria the threshold max overpressure, are mainly prepared using the outcomes of the bombardments during WWII, or outcomes of nuclear explosion test, and so they are not valid for small explosions.

We must be cautions if we use estimation criteria to predict the level of injury caused by SW of relatively small explosive charges without knowing the origin. Most probably we are using criteria that belong to large explosions, or nuclear explosions. The examples below illustrate the fact that a large explosion that produces a small peak overpressure of SW and a time-interval relatively large, creates a level of injury bigger than that of a small explosive charge for the same peak overpressure.

Example 1. (Nuclear Explosions, anti-tank mine field)
The handbook [8] shows that a nuclear explosion of power m_N (KT) can clear of an anti-tank mine field within a radius:

$$r_N = 2.46\sqrt[3]{m_{Neq}}\ , \qquad (1)$$

where mass m_{Neq} is the TNT equivalent mass of the nuclear charge m_N that goes to the creation of SW. It is known that half of the energy released during a nuclear explosion, goes for creation of SW i.e., $m_{Neq} = 0.5\ m_N$.

Let's consider a typical atomic bomb of TNT equivalent mass $m_N = 20\ KT\ TNT$, that explodes on an absolutely rigid mine-field terrain.

Find:

- The radius of the area on the anti-tank mine-field cleared during explosion.
- The max overpressure needed to explode (clear of) the antitank mines inside a circle, radius r_N.
- The time-interval of positive phase, τ.
- The impulse of SW at the estimated distance.

Solution:

a. $m_{Neq} = 0.5 \cdot 20kT = 10^7 kg.$

Using equation (1), we find the radius of the circular area that is cleared of mines:

$$r_N = 2.46\sqrt[3]{m_{Neq}} = 2.46\sqrt[3]{10^7} = 530\ m.$$

b. Using equation (1), we can write:

$$\frac{\sqrt[3]{m_{NEq}}}{r_N} = \frac{1}{k} = \frac{1}{2.46} = 0.4065 \qquad (2)$$

Substituting (2) in (3.3), i.e. in

$$\Delta P_H = 1.00 \cdot \left(\frac{\sqrt[3]{m_{NEq}}}{r_N}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{m_{NEq}}}{r_N}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{m_{NEq}}}{r_N}\right)^3,$$

we find that the peak overpressure at the distance $r_N = 530\, m$ from the detonation center is

$$\Delta P_H = 1.00 \cdot (0.4065) + 4.08 \cdot (0.4065)^2 + 12.56 \cdot (0.4065)^3 = 1.92\, KG/cm^2,$$

c. Using (3.9) we find that the time duration of SW, $r_N = 530\, m$ from the explosion center is:

$$\tau = 0.0013 \sqrt[6]{m} \cdot \sqrt[2]{r} = 0.0013 \sqrt[6]{10^7} \cdot \sqrt[2]{530} = 0.44\, sec.$$

d. Because the mines are buried in the ground, close to the surface, the dynamic impulse is zero since the flux of particles, at the positive phase, is parallel to the surface. That means that the impulse of SW is equal to the static impulse:

$$i = 266.68 \sqrt[3]{(m)^2}/r = 266.68 \sqrt[3]{(10^7)^2}/530 = 23{,}355\, P_a.sec\ .$$

Example 2.

- Use the scaling rule to find the radius r of the circular area on the same minefield that is supposed to be cleared of by a TNT conventional explosive charge of mass $m = 20\, kg$ that produces the same max overpressure, $\Delta P_H = 1.92\, KG/cm^2$, as in example 1.
- Find the time-interval, τ, the static impulse i and dynamic impulse, j, at the distance r obtained above:

Solution:

a. Substituting (2) (see Example 1) in the scaling rule,

$$\frac{\sqrt[3]{m}}{r} = \frac{\sqrt[3]{m_N}}{r_N},$$

we can write:

$$\frac{\sqrt[3]{20}}{r} = 0.4065 \ .$$

Hence, for the radius of the circular area, we find: $r = 6.68m$.

b. $$\tau = 0.0013\sqrt[6]{m} \cdot \sqrt[2]{r} = 0.0013\sqrt[6]{20} \cdot \sqrt[2]{6.68} = 0.0055\ sec.$$

The dynamic impulse of SW is equal to zero, since the particles of the compressed phase travels parallel to the ground where are placed the mines. The total impulse is equal to the static impulse:

$$i = 266.68\sqrt[3]{(\mathrm{m})^2}/r = 266.68\sqrt[3]{(20)^2}/6.68 = 294.15\ P_a.\,sec.$$

Note that the ratio of time-intervals and the ratio of impulses is respectively:

$$Ratio = 0.44/0.0055 = 80,$$

and

$$Ratio = 23{,}355/294.15 = 79.40.$$

Comment:

Note that for the same peak overpressure, $\Delta P_H = 1.92\ KG/cm^2$, the time-interval of the action on the ground of the overpressure ΔP of nuclear explosion (20 KT TNT), is 80 times greater than the time interval of the action of the overpressure of the TNT conventional charge of mass $m = 20\ kg$. The same is valid as well for the static impulse.

These big discrepancies explain the fact that for small explosive charges (same peak overpressure) we do not get the same outcome (injury/damage) as we get for nuclear explosions.
The injury/damage criteria that usually are available for nuclear explosions and large TNT charges (aviation bombs), are not valid for small explosive charges.

As a matter of fact, **the experiments, we performed long time ago**, have shown that the conventional explosive charge, that corresponds to the threshold peak overpressure $\Delta P_H = 1.92\ KG/cm^2$, is not enough to clear of the mines within the area with radius $r = 6.68\ m$. In other words, to clear of the mines, using small explosive charges, it is not sufficient to produce the threshold peak overpressure, $\Delta P_H = 1.92\ KG/cm^2$, that is needed for the nuclear explosions to detonate the mines.

The short time-length τ of the positive phase of the SW of small explosive charges, and as result the relatively small impulses, are not enough to cause the same destructive action as that of the large explosions.

In fact, to open a pathway of $r = 6.68\ m$ on the mine-field, using relatively small explosive charges, we need to detonate a TNT charge of at least

$$m = 0.0749 \cdot r^3(\Delta P_H - 1)/\beta = 0.0749 \cdot (6.68)^3(20 - 1) = 424\ Kg \text{ TNT}.$$

For small conventional charges the peak overpressure of SW needed to detonate antitank mines is $\Delta P_H = 20\ KG/cm^2$. (see Example 4, section 3.1).

We have to outline that the peak-overpressure (or/and impulse) is used to estimate the level of injury/damage since, in general, there are no means to measure the overpressure/impulse at the time-interval of SW.
An interesting and effective method to predict the level of injury is developed in [5]. In that method they circumvent the fact that are no clear criteria on the prediction of the level of injury/damage [5, page 3: Damage/injury criteria].

5.2 Lethal Action of Explosives on Human Being

We will see only injuries caused to the human beings by the primary action of SW. According to Orlenko [1, p.206], the eardrums have a very small period of self-oscillations T. For that reason, the lethal action of the SW related to the injury of eardrums can be estimated using as criterion the peak-overpressure ΔP_H.

The incapacitation of lung, that is another vulnerable organ, can be estimated as well using the peak-overpressure criterion.

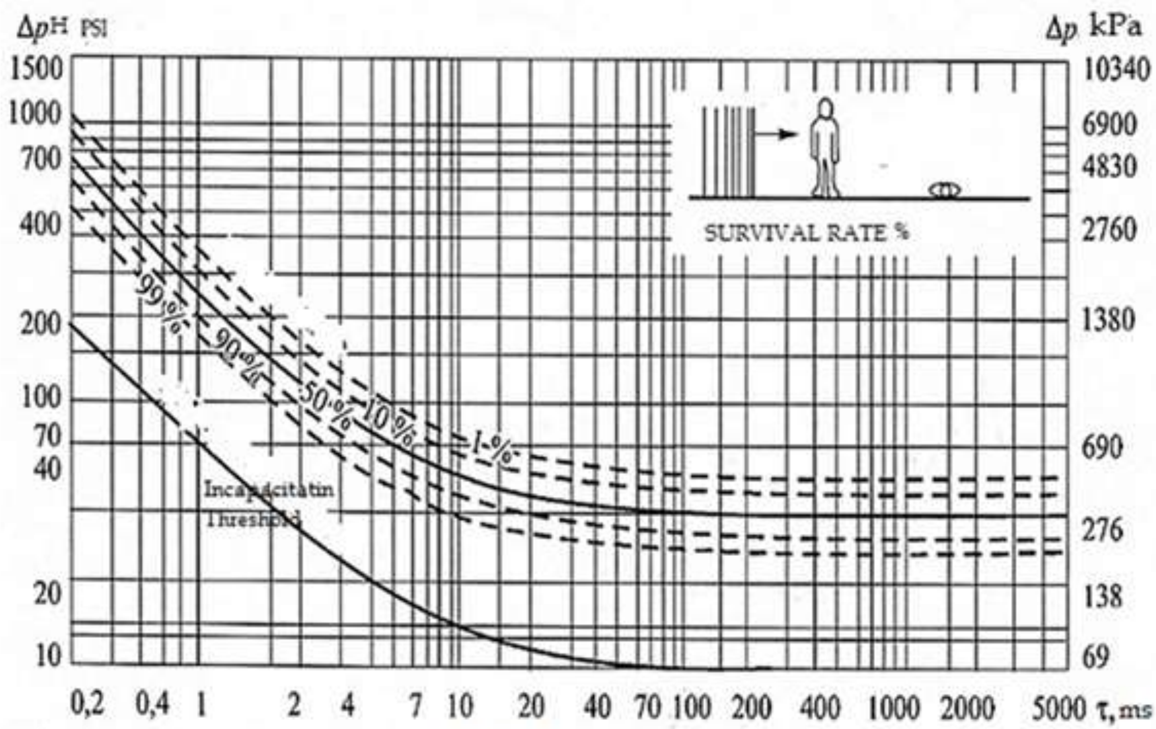
The graph in Fig. "Little Man" represents the ratio of survivability of a 70 kg man/woman as a function of time-interval τ of SW and the peak overpressure ΔP_H [9], [4].

At the same time the graph shows the lethality probability of that man, since, for example, 90% survivability is equal to 10% lethality (incapacitation) resulting from lung and/or eardrum injury. Note that τ is in milliseconds (ms).

The lethal probability of lungs and, or eardrums is a conditional probability. It represents the probability of injury given the time-length (and the related peak overpressure) of SW that interreacts with human body, i.e. $P_\tau(I)$.

Some Observations Related to the "Little Man" Graph

1. The graph "Little Man" shows that for those SWs that have a time-interval $\tau > 50\,ms$, the 50% incapacitation probability results when the peak overpressure is around $\Delta P_H = 50\,psi = 3.52\,KG/cm^2$. The peak overpressure threshold does not depend on time-interval τ for those large explosive charges (or nuclear ones), i.e. it remains constant and equal to around $3.50\,KG/cm^2$. That is the reason that for nuclear explosions there are lethality/destruction criteria that do not depend on the TNT equivalent mass of the explosive charge.



Little Man

2. When τ decreases, $\tau < 50\,ms$, the peak overpressure that causes 50% incapacitation probability increases continuously, starting from around $\Delta P_H = 50\,psi = 3.50\,KG/cm^2$.
For example, when $\tau = 1.5\,ms = 0.0015\,sec$, to have 50% incapacitation probability the peak overpressure must be at least $\Delta P_H = 150\,psi = 10.55\,KG/cm^2$.

We observe the same pattern for any other incapacitation probability (1%, 10%, 90%, 99%).

Estimating Incapacitation Probability, $\boldsymbol{P_\tau(i)}$. Ground Surface Explosion

To predict the mass m of an explosive charge and the distance r related to a certain time-interval τ, using the graph in Fig. "Little Man", we have to employ the equations (3.3), or (3.5), i.e.:

- $\Delta P_H = 1.00 \cdot (\frac{\sqrt[3]{m}}{r}) + 4.08 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3,$ (5.1)
- $\Delta P_H = 1.27 \cdot (\frac{\sqrt[3]{m}}{r}) + 2.40 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 12.10 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3 - 0.0196$ (5.2)

valid for $0.1\ KG/cm^2 \leq \Delta P_H \leq 10\ KG/cm^2$, and the equation (3.4),

- $\Delta P_H = 13.84 \cdot m/r^3 + 1$, (5.3)

valid for $\Delta P_H \geq 10\ KG/cm^2$.

The time-interval (duration of positive phase), equation (3.9):

$$\tau = 0.0013\sqrt[6]{m} \cdot \sqrt[2]{r}. \quad (5.4)$$

a. Fifty Percent Incapacitation Probability

Peak overpressure $0.1\ KG/cm^2 \leq \Delta \boldsymbol{P_H} \leq 10\ KG/cm^2$

Assume that we need to find the TNT equivalent mass m that produces a max overpressure ΔP_H at a distance r from the center of explosion, as well as a given time-interval of τ. We consider that $0.1\ KG/cm^2 \leq \Delta P_H \leq 10\ KG/cm^2$.

Solving equation (5.1) for $\sqrt[3]{m}/r$, we find a number that we denote by k, i.e.

$$\sqrt[3]{m}/r = k. \quad (5.5)$$

Hence, we find the distance r, where is observed the max overpressure ΔP_H:

$$r = \frac{1}{k}\sqrt[3]{m}. \quad (5.6)$$

Substituting (5.6) in (5.4), we find the relation between time-interval τ and mass of the explosive charge:

$$\tau = 0.0013 \cdot \sqrt[3]{m}/\sqrt{k}. \quad (5.7)$$

Solving the last equation for m we have:

$$m = (\frac{\tau}{0.0013})^3 \cdot k^{3/2}. \tag{5.8}$$

Examples below, illustrate the way we estimate the mass m of an explosive charge that produces a given incapacitation probability $\boldsymbol{P_\tau(I)}$ and the corresponding distance r from the center of explosion, using the time-interval τ and peak overpressure ΔP_H.

Example 1.

The 50% probability of incapacitation, $P_\tau(i) = 0.50$, of the personnel exposed to SW of a surface-ground explosion, corresponds to a SW with time-interval $\tau = 90\ ms = 0.090\ sec$ and to a peak overpressure $\Delta P_H = 40\ psi = 2.8\ KG/cm^2$ (See the graph "Little Man").

Find the corresponding mass m of the explosive charge that produces the given peak overpressure as well as the related distance r from the center of explosive.

Solution:

Substituting in (5.3), we have:

$$2.8 = 1.00 \cdot (\frac{\sqrt[3]{m}}{r}) + 4.08 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3,$$

Solving the above equation, we find $k = \sqrt[3]{m}/r = 0.479$. Hence, $r = \frac{1}{0.479}\sqrt[3]{m}$.

Using (5.8), we find:

$$m = (\frac{0.090}{0.0013})^3 \cdot (0.479)^{3/2} = 110{,}002\ \ kg\ TNT.$$

The related distance is

$$r = \frac{1}{k}\sqrt[3]{m} = \frac{1}{0.48}\sqrt[3]{110{,}346.7} = 100\ m.$$

As we see, to have a (50%) incapacitation probability it is needed a large explosive charge of at least around 110,000 Kg TNT equivalent.

Note that in order to achieve the required 50% incapacitation probability for the SWs with very short time-length we need to have a very large max overpressure.

b. Fifty Percent Incapacitation Probability, $\boldsymbol{\Delta P_H} > 10\ KG/cm^2$.

We will show an example to demonstrate the way we estimate the mass of explosive charge and time interval of SW needed to produce a given incapacitation probability $\boldsymbol{P_\tau(i)}$ of people exposed to SW.

Example 2.

The 50% probability of incapacitation of the personnel, i.e. $P_\tau(I) = 0.50$, corresponds to an explosion with time-interval $\tau = 1\ ms = 0.001\ sec.$, and to a peak overpressure $\Delta P_H = 250\ psi = 17.58\ KG/cm^2$ (See the graph Fig. "Little Man").

Find the corresponding mass m of the explosive charge that creates the given peak overpressure, $\Delta P_H = 17.58\ KG/cm^2$ as well as the related distance r from the center of explosion.

Solution:

Since $\Delta P_H > 10KG/cm^2$ we use equation (5.3).

Substituting $\Delta P_H = 17.58\ KG/cm^2$ in (5.3), we can write:

$$17.58 = 13.84 \cdot m/r^3 + 1.$$

Hence, we have:

$$m = \frac{17.58-1}{13.84} \cdot r^3 = 1.198 \cdot r^3.$$

Substituting $\tau = 0.001$, $m = 1.198 \cdot r^3$ in (5.4), we can write:

$$0.001 = 0.0013\sqrt[6]{1.198} \cdot r.$$

Hence, we find the distance from the center of explosion

$$r = 0.75\ m.$$

For the mass m of explosive charge we have:

$$m = 1.198 \cdot r^3 = 1.198 \cdot 0.75^3 = 0.50\ kg.$$

Example 3.

A 100 kg TNT explosive charge, is detonated on a flat rigid ground. Find the incapacitation probability at a distance:

a. 10 meters.
b. 5 meters from the center of explosion.

Solution:

a. The solution is similar to the one in Example 1.

$$k = \sqrt[3]{m}/r = \sqrt[3]{100}/10 = 0.464.$$

$$\Delta P_H = 1.00 \cdot (0.465) + 4.08 \cdot (0.464)^2 + 12.56 \cdot (0.464)^3 = 2.597\ KG/cm^2 = 37\ psi\ .$$

$$\tau = 0.0013 \cdot \sqrt[3]{m}/\sqrt{k} = 0.0013 \cdot \sqrt[3]{100}/\sqrt{0.464} = 0.00885.$$

The "Little man" graph shows that for $\tau = 0.00885$ and $\Delta P_H = 37\ psi$, the incapacitation probability is between zero, and 1%, $P_\tau(i) = 0.00\ or\ 0.01$.
No serious injury resulting from SW. Maybe a temporary loss of hearing.

Note: For the given explosive charge, at distance 10 meters, the incapacitation, most likely, might be due to the flying debris, or fragments of the metallic body of ammunition.

b. Incapacitation probability at a distance 5 meter from the center of explosion.

$$k = \sqrt[3]{m}/r = \sqrt[3]{100}/5 = 0.928.$$

$$\Delta P_H = 1.00 \cdot (0.928) + 4.08 \cdot (0.928)^2 + 12.56 \cdot (0.928)^3 = 14.49 = 206.1\ psi.$$

Since $\Delta P_H > 10 KG/cm^2$ we have to use a similar approach as in Example 2.

Substituting in (5.3), $\Delta P_H = 14.49\ KG/cm^2$, $m = 100\ kg$, $r = 5\ m$, we find:

$$\Delta P_H = 13.84 \cdot 100/5^3 + 1 = 12.072\ KG/cm^2 = 172\ psi\ .$$

$$\tau = 0.0013\sqrt[6]{m} \cdot \sqrt[2]{r} = \tau = 0.0013\sqrt[6]{100} \cdot \sqrt[2]{5} = 0.0063\ sec = 6.3\ ms\ .$$

The "Little Man" graph shows that for the above values of max overpressure and time-interval, incapacitation probability is $P_\tau(i) = 100\%$.

Example 4.
A man standing next to a brick wall is hit by a SW generated by an explosive charge.

1. Find the mass m of the explosive charge needed to be detonated on the ground

at a standoff distance of $r = 15\ m$, from the wall if we want to have a reflected peak overpressure of $\Delta P_r = 1.5\ KG/cm^2$.

2. Find the incapacitation probability that the man standing next to the brick wall will be a casualty.

Solution:
We consider the coefficient β that corresponds to the energy of SW lost during the reflection from the wall to be $\beta = 0.80$.

1. Substituting $\Delta P_r = 1.50\ KG/cm^2$in (4.1), we can write:

$$2\Delta P_H + \frac{6(\Delta P_H)^2}{\Delta P_H + 7} = 1.50$$

Solving the above equations for ΔP_H, we find: $\Delta P_H = 0.605\ KG/cm^2$.

Substituting $\Delta P_H = 0.605$, in (3.6), we have:

$$1.00 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{\beta m}}{r}\right)^3 = 0.605.$$

Solving for $\sqrt[3]{\beta m}/r$, we find: $\sqrt[3]{\beta m}/r$=0.23.

Hence:

$$\sqrt[3]{0.80m}/15 = 0.23.$$

and

$$m = 15.21\ kg\ TNT.$$

2. To find the incapacitation probability of the standing man, exposed to the reflected peak overpressure $\Delta P_r = 1.50 \frac{KG}{cm^2} = 21.43\ psi$, we need to find the time duration τ of the reflected SWr.

We consider that the peak overpressure $\Delta P_r = 1.50\ KG/cm^2$ of SWr, is created by the reflected SW of a double mass, $(2\beta m) = 2 \cdot 0.80 \cdot 15.21 = 24.34\ kg\ TNT$, exploded at the distance $r = 15\ m$.

That assumption is based on the physics model that the reflected wave originates at the virtual center of explosion, located on the opposite side of the reflective surface, at the same distance $r = -15\ m$.

Using (5.4), we find the time-interval

$$\tau = 0.0013\sqrt[6]{m} \cdot \sqrt[2]{r} = 0.0013\sqrt[6]{24.34} \cdot \sqrt[2]{15} = 0.0086\ s = 8.6\ ms.$$

Using the "Little Man" graph, we find that the point with coordinates $\tau = 8.6\ ms$ and $\Delta P_H = 21.43\ psi$ is between the curve "Incapacitated Threshold" and the curve "1% casualty = 99% survivability".

The standing man will survive without any serious injury.

Example 5.

The 50% probability of incapacitation, $P_\tau(i) = 0.50$, of the personnel exposed to SW of an explosion, corresponds to a SW with time-interval $\tau = 4\ ms = 0.004\ sec$ and to a peak overpressure $\Delta P_H = 90\ psi = 6.33\ KG/cm^2$ (See the graph "Little Man"). Find the corresponding mass m of the explosive charge that produces the given peak overpressure as well as the related distance r from the center of explosion. Consider a ground surface burst, and a rigid ground.

Solution:

Substituting in (5.3), we have:

$$1.00 \cdot \left(\frac{\sqrt[3]{m}}{r}\right) + 4.08 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 12.56 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3 = 6.33,$$

Solving the above equation, we find $k = \sqrt[3]{m}/r = 0.672$. Hence, $r = \frac{1}{0.672}\sqrt[3]{m}$.

Using (5.8), we find:

$$m = (\frac{0.004}{0.0013})^3 \cdot (0.672)^{\frac{3}{2}} = 16.05\ \ kg \text{ TNT}.$$

The related distance is

$$r = \frac{1}{k}\sqrt[3]{m} = \frac{1}{0.672}\sqrt[3]{16.05} = 3.75\ m.$$

As we see, to have (50%) incapacitation probability it is needed a small explosive charge, at least around 16.05 Kg TNT equivalent.

Note that in order to achieve the required 50% incapacitation probability for a SW with very short time-length τ we need to have a very large max overpressure, $\Delta P_H = 90\ psi = 6.33\ KG/cm^2$.

Human Tolerance to the SW

Human tolerance to the SW of an explosion and the level of injury mainly depends on:

- the positioning of the body (standing, prone, etc.) with respect to SW.
- overpressure and the peak overpressure of incident SW and/or reflected SWr.
- impulse of SW as well as the time-interval of positive phase of overpressure
- characteristics of terrain and the distribution of objects (targets, shelters, structures, etc.).
- Age and mass of the person.

As such, the level of injury can be measured using the probability that a random person will be incapacitated by the action of SW.

The model that is used to estimate the probability of injury is an ideal one and can serve as a reference indicator.

An important factor used to determine the probability level of injury is the time-duration of the positive phase of SW. As the "Little Man" graph shows the human tolerance to the SW overpressure is higher for short-time duration (relatively small explosions, small explosive charges) than that of long duration SW overpressure (relatively big explosions).

For example, the graph of "little man", shows that at a 50% incapacitation probability, to the time-interval $\tau = 4\ ms = 0.004\ sec$ corresponds a peak overpressure of approximately $\Delta P_H = 90\ psi = 6.33\ \ KG/cm^2$, while to the time interval $\tau = 90\ ms = 0.090\ sec$ corresponds a peak overpressure of around $\Delta P_H = 40\ psi = 2.8\ KG/cm^2$.

According to [10], man's response to a short-time-interval τ, $3ms < \tau < 5\ ms$, of the SW peak overpressure, corresponds to the threshold lethality level (casualty probability zero) of a pressure 30 – 40 psi ($2.1 - 2.8\ KG/cm^2$).

5.3 Total Incapacitation Probability

The probability level of an injury depends on the time-duration τ of the positive overpressure of the SW as well as on the overpressure. As a matter of fact, the probability level of an injury can be predicted using the peak overpressure and the specific impulse which depends on τ. A SW with short-time duration τ has a higher peak overpressure than a SW of long-time duration.

To estimate the level of injury at humans, we consider the peak overpressure and the time-interval of incident SW (or reflected SW), and use the graph "Little Man".

As we illustrated above and as the "Little Man" shows, for relatively small explosive charges, (with time-interval $\tau < 50\ ms$), the incapacitation probability, and the related peak overpressure, should be estimated experimentally for each explosive charge, or at least for some most significative charges. The criteria for the estimation of injuries that are shown in many papers, are based on the nuclear explosions, or on the WWII bombardments. They must not be used for the SW of short time duration.

Most likely, this is the reason that, in the contemporary literature, in general, we do not find criteria to estimate the level of injury/damage caused by relatively small conventional charges. Nevertheless, those criteria are used also for relatively small conventional explosive charges.

A good source of information on the level of injury/lethality of human, for relatively small conventional explosive charges we referred to, are the US Manual [10], and [9]. The data of the USA manual (page 11-12) for small explosive charges, (duration 3 – 5 ms) are a confirmation of the validity of our demonstration on the level of injury/lethality of people exposed to SW of small conventional ammunitions.

Total Probability

Let's estimate the total probability that corresponds to the time duration $\tau = 4\ ms = 0.004\ sec$. In table 1, it is given the distribution of the conditional probability as a function of the peak overpressure that corresponds to $\tau = 4\ ms = 0.004\ sec$ and to a TNT charge of mass $m = 16.05\ kg$.

Table 1. Conditional Incapacitation Probability, P_i

$\Delta P_H(\ psi)$	-	130	115	90	80	60	28
$\Delta P_H(\ KG/cm^2)$	-	9.14	8.08	6.328	5.625	4.22	1.96
P(i)	100%	99%	90%	50%	10%	1%	0%
$k = \sqrt[3]{m}/r$	-	0.777	0.741	0.672	0.641	0.57	0.41
$r = \sqrt[3]{m}/\mathrm{k}$	0	3.25	3.40	3.75	3.94	4.43 m	6.21

The mass $m = 16.05\ kg$ is determined in the above Exercise 5.

To predict the total incapacitation probability, we will use the same method we followed in the paper, described in details at [11].

Using data of table 1, we will only show the approximate calculation of the total probability:

$P(I) \approx \frac{\pi}{D}\sum(r_{i+1}^2 - r_i^2)p(\bar{r}_i) = (3.25^2 - 0^2)\cdot(0.99 + 1)/2 + (3.40^2 - 3.25^2)\cdot(0.90 + 0.99)/2 + (3.75^2 - 3.40^2)\cdot(0.50 + 0.90)/2 + (3.94^2 - 3.75^2)\cdot(0.10 + 0.50)/2 + (4.43^2 - 3.94^2)\cdot(0.10 + 0.01)/2 + (6.21^2 - 4.43^2)(0.00 + 0.01)/2 = 13.868\frac{\pi}{D} = 13.868\cdot\pi/(\pi\cdot 6.21^2) = 0.36.$

Thus, $P(I) \approx 0.36.$

Expected value of Casualties is

$$E(I) = N\cdot\left(13.868\cdot\frac{\pi}{D}\right) = \frac{N}{D}\cdot(13.86\cdot\pi) = n\cdot(13.86\cdot\pi),$$

where: N is the number of personnel in the flat circular area $D = \pi\cdot R^2$, $n = N/D$ is the density of personnel at the area D with radius $R = 6.21\ m$, in our case (see table 1, last column).

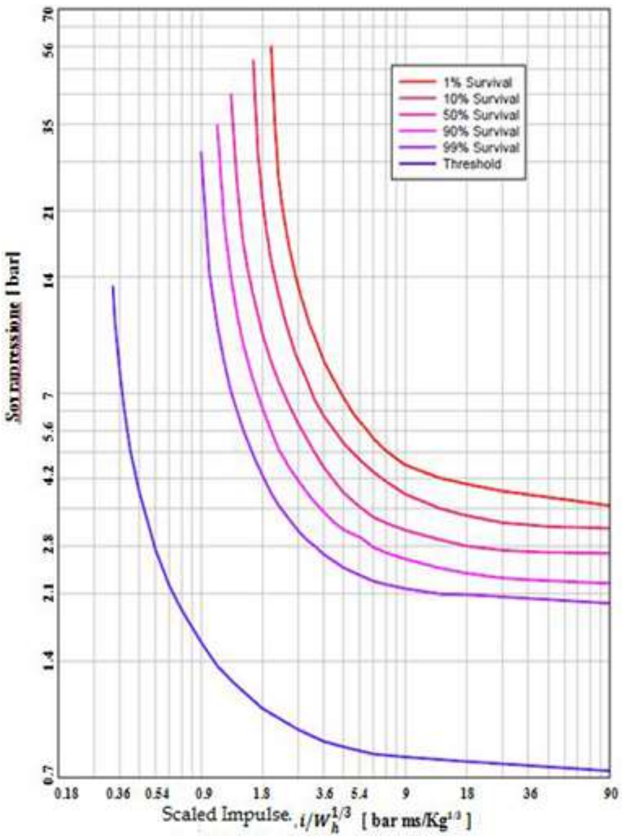
Thus, for a density $n = 0.04\ persons/m^2$, i.e. 4 persons randomly distributed inside the area of 100 m^2, on average, it will be incapacitated/injured

$$E(I) = n(13.86\cdot\pi) = 0.04(13.86\cdot\pi) = 1.74 \text{ persons.}$$

Lung and Eardrum Injury

The breath system and the hearing system are the most vulnerable organs of the human beings. We will illustrate the level of casualty/survival using the following exercises.

Example 6 (Lung Injury)



Assess Lung Injury caused by the SW of the surface-ground explosion of a mass m= 100 lb. = 45.3592 kg. TNT at a distance r = 100 ft. = 30.48 m. Mass of human body $w_h = 60\ kg$, altitude: sea level [12].

Note. In the above figure it is given the level of survival of lungs that at the same time is the probability of survival of a person standing at a certain distance r from the detonation center [10], [13].

The level of injury depends as well on the mass w_h of the person exposed to SW.

Solution

First, we find the peak overpressure that corresponds to

$$k = \sqrt[3]{m}/r = \sqrt[3]{45.36}/30.50 = 0.117.$$

Substituting in (3.3), we have:

$$\Delta P_H = 1.00 \cdot (0.117) + 4.08 \cdot (0.117)^2 + 12.56 \cdot (0.117)^3 = 0.193kg/= 0.189bar.$$

The specific impulse is

$$i = \frac{266.70\sqrt[3]{m^2}}{r} = \frac{266.70\sqrt[3]{45.36^2}}{30.48} = 111.3\ Pa.s = 1.113\ bar\ ms$$

Scaled impulse is

$$i_s = i/w_h^{1/3} = 1.113/60^{1/3} = 0.284\ bar\ ms/kg^{1/3}.$$

Using the graph below, we see that the point with coordinates ($i_s = 0.284$) and ($\Delta P_H = 0.189$ bar) is below the threshold curve (100% survival).

So, there is no serious injury.

Example 7 (Eardrum Injury)

Find the probability of eardrum injury caused by the SW of an air-burst of mass m= 100 lb. = 45.3592 kg. TNT at a distance r = 100 ft. =30.48 m.

Solution:

First, we find the peak overpressure that corresponds to

$$k = \sqrt[3]{m}/r = \sqrt[3]{45.36}/30.50 = 0.117.$$

Substituting in (2.6), i.e. in

$$\Delta P_H = 0.79 \cdot (\frac{\sqrt[3]{m}}{r}) + 2.57 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^2 + 6.28 \cdot \left(\frac{\sqrt[3]{m}}{r}\right)^3$$

We find the peak overpressure of incident SW

$$\Delta P_H = 0.79 \cdot (0.117) + 2.57 \cdot (0.117)^2 + 6.28 \cdot (0.117)^3 = 0.138\frac{KG}{cm^2} = 1.97psi.$$

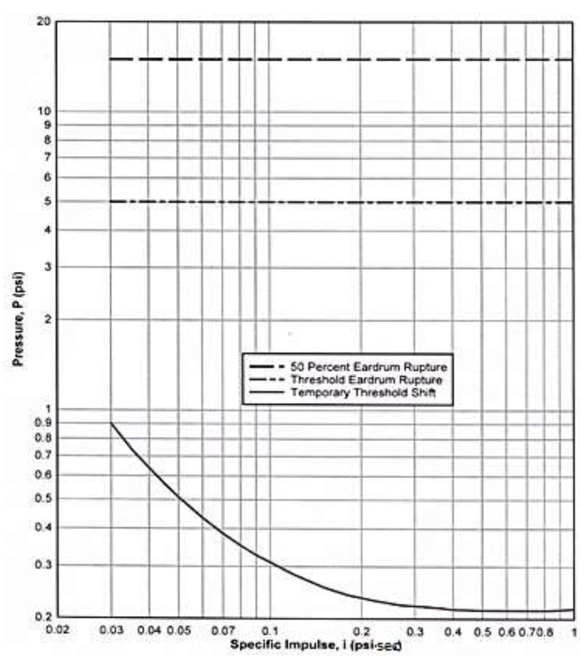
Specific Impulse

Substituting in (2.8), we find the specific impulse:

$$i = 168\sqrt[3]{m^2}/r = 168\sqrt[3]{45.36^2}/30.50 = 70.6 \text{ Pa} \cdot s = 1.016 \times 10^{-2} psi\ s.$$

Level of Eardrum Injury

Using this graph, we see that the point with coordinates ($i_s = 1.016 \times 10^{-2} psi\ s$)



and ($\Delta P_H = 1.97\ psi$) is above the temporary threshold shift, but below the curve of 50% Eardrum Rupture.
So, there is no serious ear injury, but only a temporary hearing loss.

Instead of Conclusions

In this long paper on the shock wave of explosion, we tried to explore to the reader some basic concepts, laws and equations to solve practical problems related to the Physics of Explosion, and in particular those related to the shock wave.

The topics we included in the material are developed using the laws and formulas of different authors. We reached almost the same conclusions or results, using approaches different from many other authors in the area of SW. We compared our results obtained in problem solving with some well-known fundamental outcomes generally accepted in the studies of SW.

We demonstrated through problem-solving that the outcomes related to the SW are approximate, mostly indicative solutions obtained in ideal or simplified conditions.

We illustrated that the level of injury/damage caused by the interaction of an object (soft or hard target) with the SW of explosion depends on the power of explosion, and so we do not have general evaluation criteria valid for all SW of different peak overpressures and time-durations. In other words, the criteria, generally used to classify the damage caused by the shock wave of huge explosions are not valid for relatively small explosions.

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